GCE MARKING SCHEME

MATHEMATICS
AS/Advanced

JANUARY 2012

INTRODUCTION

The marking schemes which follow were those used by WJEC for the January 2012 examination in GCE MATHEMATICS. They were finalised after detailed discussion at examiners' conferences by all the examiners involved in the assessment. The conferences were held shortly after the papers were taken so that reference could be made to the full range of candidates' responses, with photocopied scripts forming the basis of discussion. The aim of the conferences was to ensure that the marking schemes were interpreted and applied in the same way by all examiners.

It is hoped that this information will be of assistance to centres but it is recognised at the same time that, without the benefit of participation in the examiners' conferences, teachers may have different views on certain matters of detail or interpretation.

WJEC regrets that it cannot enter into any discussion or correspondence about these marking schemes.

Mathematics C1

1. (*a*) (i) Gradient of $AB(CD) = \frac{\text{increase in } y}{\text{increase in } x}$ M1

Gradient of $AB = -2$, gradient of $CD = -2$,
(or equivalent, at least one correct) A1

Gradient of AB = gradient of $CD \Rightarrow AB$ and CD are parallel
(c.a.o.) A1

(ii) A correct method for finding the equation of AB using candidate's gradient for AB M1

Equation of AB: $y - 14 = -2[x - (-5)]$ (or equivalent)
(f.t. candidate's gradient for AB) A1

(iii) Use of gradient L × gradient $AB = -1$ M1

$\left\{\begin{array}{l}\text{A correct method for finding the equation of } L \text{ using} \\ \text{candidate's gradient for } L\end{array}\right\}$ (M1)

(to be awarded only if corresponding M1 is not awarded in part (ii))

Equation of L: $y - 8 = {}^1/_2(x - 3)$ (or equivalent)
(f.t. candidate's gradient for AB) A1

Equation of L: $x - 2y + 13 = 0$ (convincing) A1

Note: Total mark for part (*a*) is 8 marks

(*b*) (i) An attempt to solve equations of AB and L simultaneously M1

$x = -1$, $y = 6$ (c.a.o.) A1

(ii) A correct method for finding the coordinates of the mid-point of AB M1

Mid-point of AB has coordinates $(-2, 8)$ A1

A correct method for finding the length of EF M1

$EF = \sqrt{5}$ (f.t. the candidate's derived coordinates for E and F)
A1

2. (*a*) $\frac{9 + 4\sqrt{2}}{5 + 3\sqrt{2}} = \frac{(9 + 4\sqrt{2})(5 - 3\sqrt{2})}{(5 + 3\sqrt{2})(5 - 3\sqrt{2})}$ M1

Numerator: $45 - 27\sqrt{2} + 20\sqrt{2} - 24$ A1

Denominator: $25 - 18$ A1

$\frac{9 + 4\sqrt{2}}{5 + 3\sqrt{2}} = 3 - \sqrt{2}$ (c.a.o.) A1

Special case

If M1 not gained, allow B1 for correctly simplified numerator or denominator following multiplication of top and bottom by $5 + 3\sqrt{2}$

(*b*) $\sqrt{8} \times \sqrt{10} = 4\sqrt{5}$ B1

$\frac{\sqrt{90}}{\sqrt{2}} = 3\sqrt{5}$ B1

$\frac{30}{\sqrt{5}} = 6\sqrt{5}$ B1

$(\sqrt{8} \times \sqrt{10}) + \frac{\sqrt{90}}{\sqrt{2}} - \frac{30}{\sqrt{5}} = \sqrt{5}$ (c.a.o.) B1

3. y-coordinate of $P = 7$ B1

$\frac{dy}{dx} = 4x - 8$

(an attempt to differentiate, at least one non-zero term correct) M1

An attempt to substitute $x = 3$ in candidate's expression for $\frac{dy}{dx}$ m1

Value of $\frac{dy}{dx}$ at $P = 4$ (c.a.o.) A1

Gradient of normal = $\dfrac{-1}{\text{candidate's value for } \frac{dy}{dx}}$ m1

Equation of normal to C at P: $y - 7 = -{}^1/_4(x - 3)$ (or equivalent)

(f.t. candidate's value for $\frac{dy}{dx}$ and the candidate's derived y-value at $x = 3$

provided M1 and both m1's awarded) A1

4. (*a*) $\left(x + \frac{3}{x}\right)^4 = x^4 + 4x^3\left(\frac{3}{x}\right) + 6x^2\left(\frac{3}{x}\right)^2 + 4x\left(\frac{3}{x}\right)^3 + \left(\frac{3}{x}\right)^4$

(all terms correct B2)

(3 or 4 terms correct B1)

$\left(x + \frac{3}{x}\right)^4 = x^4 + 12x^2 + 54 + \frac{108}{x^2} + \frac{81}{x^4}$

(all terms correct B2)

(3 or 4 terms correct B1)

(– 1 for further incorrect simplification)

(*b*) ${}^nC_2 \times 2^k = 760$ ($k = 1, 2$) M1

Either $2n^2 - 2n - 760 = 0$ or $n^2 - n - 380 = 0$ or $n(n - 1) = 380$ A1

$n = 20$ (c.a.o.) A1

5. (*a*) $a = 3$ B1

$b = -1$ B1

$c = 2$ B1

(*b*) An attempt to substitute 1 for x in an appropriate quadratic expression

(f.t. candidate's value for b) M1

Maximum value = ${}^1/_8$ (c.a.o.) A1

6. An expression for $b^2 - 4ac$, with at least two of a, b, c correct M1

$b^2 - 4ac = 4^2 - 4 \times (k + 6) \times (k + 3)$ A1

Putting $b^2 - 4ac < 0$ m1

$k^2 + 9k + 14 > 0$ (convincing) A1

Finding critical values $k = -7, k = -2$ B1

A statement (mathematical or otherwise) to the effect that

$k < -7$ or $-2 < k$ (or equivalent)

(f.t. only critical values of ± 7 and ± 2) B2

Deduct 1 mark for each of the following errors:

the use of non-strict inequalities

the use of the word 'and' instead of the word 'or'

7. (*a*) $y + \delta y = 8(x + \delta x)^2 - 5(x + \delta x) - 6$ B1

Subtracting y from above to find δy M1

$\delta y = 16x\delta x + 8(\delta x)^2 - 5\delta x$ A1

Dividing by δx and letting $\delta x \to 0$ M1

$\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{\delta y}{\delta x} = 16x - 5$ (c.a.o.) A1

(*b*) $\frac{dy}{dx} = a \times (-1) \times x^{-2} + 10 \times \frac{1}{2} \times x^{-1/2}$ B1, B1

Attempting to substitute $x = 4$ in candidate's expression for $\frac{dy}{dx}$ and putting expression equal to 3 M1

$a = -8$ (c.a.o.) A1

8. (*a*) Use of $f(3) = 35$ M1

$27a - 63 - 10 = 35 \Rightarrow a = 4$ (convincing) A1

(*b*) Attempting to find $f(r) = 0$ for some value of r M1

$f(-2) = 0 \Rightarrow x + 2$ is a factor A1

$f(x) = (x + 2)(4x^2 + ax + b)$ with one of a, b correct M1

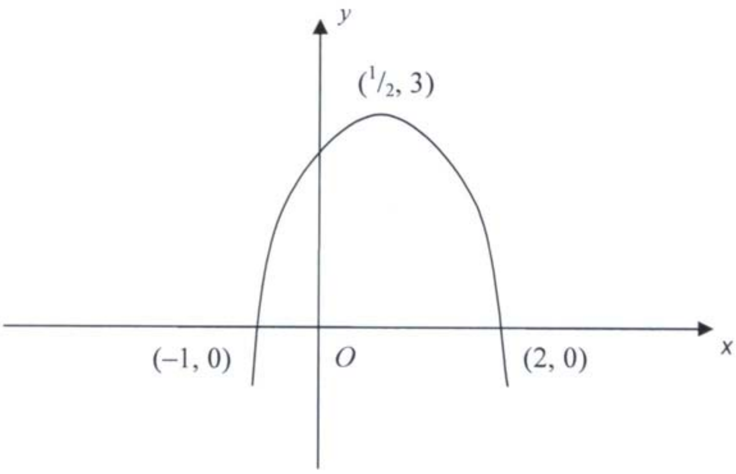
$f(x) = (x + 2)(4x^2 - 8x - 5)$ A1

$f(x) = (x + 2)(2x + 1)(2x - 5)$ (f.t. only $4x^2 + 8x - 5$ in above line) A1

Special case

Candidates who, after having found $x + 2$ as one factor, then find one of the remaining factors by using e.g. the factor theorem, are then awarded B1 instead of the final three marks

9. (*a*)

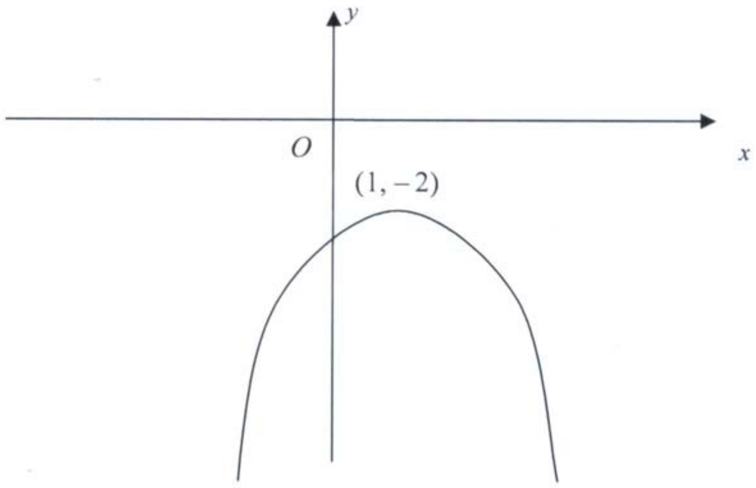


Concave down curve with y-coordinate of maximum = 3 B1

x-coordinate of maximum = $^1/_2$ B1

Both points of intersection with x-axis B1

(*b*)



(i) Concave down curve with x-coordinate of maximum = 1 B1
Graph below x-axis and y-coordinate of maximum = – 2 B1

(ii) No real roots (f.t. the number of times the candidate's curve cuts the x-axis) B1

10. (*a*) $\frac{dy}{dx} = 3x^2 - 12x + 12$ B1

Putting derived $\frac{dy}{dx} = 0$ M1

$3(x-2)^2 = 0 \Rightarrow x = 2$ A1

$x = 2 \Rightarrow y = -1$ (c.a.o) A1

(*b*) **Either:**

An attempt to consider value of $\frac{dy}{dx}$ at $x = 2^-$ and $x = 2^+$ M1

$\frac{dy}{dx}$ has same sign at $x = 2^-$ and $x = 2^+ \Rightarrow (2, -1)$ is a point of inflection A1

Or:

An attempt to find value of $\frac{d^2y}{dx^2}$ at $x = 2$, $x = 2^-$ and $x = 2^+$ M1

$\frac{d^2y}{dx^2} = 0$ at $x = 2$ and $\frac{d^2y}{dx^2}$ has different signs at $x = 2^-$ and $x = 2^+$

$\Rightarrow (2, -1)$ is a point of inflection A1

Or:

An attempt to find the value of y at $x = 2^-$ and $x = 2^+$ M1

Value of y at $x = 2^- < -1$ and value of y at $x = 2^+ > -1 \Rightarrow (2, -1)$ is a point of inflection A1

Or:

An attempt to find values of $\frac{d^2y}{dx^2}$ and $\frac{d^3y}{dx^3}$ at $x = 2$ M1

$\frac{d^2y}{dx^2} = 0$ and $\frac{d^3y}{dx^3} \neq 0$ at $x = 2 \Rightarrow (2, -1)$ is a point of inflection A1

Mathematics C2

1.

1	0·5		
1·5	0·674234614		
2	0·828427124		
2·5	0·968564716	(5 values correct)	B2
3	1·098076211	(3 or 4 values correct)	B1

Correct formula with $h = 0·5$ M1

$I \approx \frac{0·5}{2} \times \{0·5 + 1·098076211 + 2(0·674234614 + 0·828427124 + 0·968564716)\}$

$I \approx 6·540529119 \div 4$

$I \approx 1·63513228$

$I \approx 1·635$ (f.t. one slip) A1

Special case for candidates who put $h = 0·4$

1	0·5		
1·4	0·641255848		
1·8	0·768691769		
2·2	0·885939445		
2·6	0·995233768		
3	1·098076211	(all values correct)	B1

Correct formula with $h = 0·2$ M1

$I \approx \frac{0·4}{2} \times \{0·5 + 1·098076211 + 2(0·641255848 + 0·768691769 + 0·885939445 + 0·995233768)\}$

$I \approx 8·180317871 \div 5$

$I \approx 1·636063574$

$I \approx 1·636$ (f.t. one slip) A1

Note: Answer only with no working shown earns 0 marks

2. (*a*) $10(1 - \cos^2\theta) + 7\cos\theta = 5\cos^2\theta + 8$

(correct use of $\sin^2\theta = 1 - \cos^2\theta$) M1

An attempt to collect terms, form and solve quadratic equation in $\cos\theta$, either by using the quadratic formula or by getting the expression into the form $(a\cos\theta + b)(c\cos\theta + d)$, with $a \times c$ = candidate's coefficient of $\cos^2\theta$ and $b \times d$ = candidate's constant m1

$15\cos^2\theta - 7\cos\theta - 2 = 0 \Rightarrow (3\cos\theta - 2)(5\cos\theta + 1) = 0$

$\Rightarrow \cos\theta = \frac{2}{3}, \quad \cos\theta = -\frac{1}{5}$ (c.a.o.) A1

$\theta = 48·19°, 311·81°$ B1

$\theta = 101·54°, 258·46°$ B1 B1

Note: Subtract 1 mark for each additional root in range for each branch, ignore roots outside range.

$\cos\theta$ = +, –, f.t. for 3 marks, $\cos\theta$ = –, –, f.t. for 2 marks

$\cos\theta$ = +, +, f.t. for 1 mark

(*b*) $x - 50 = -43°, 223°, 317°$ (at least one value) B1
$x = 7°, 273°$, (both values) B1 B1

Note: Subtract from final 2 marks 1 mark for each additional root in range, ignore roots outside range.

(*c*) $\sin\phi \le 1, \cos\phi \le 1$ and thus $\sin\phi + \cos\phi \le 2$ E1

3. (*a*) $\frac{1}{2} \times x \times (2x - 3) \times \sin 150° = 6{\cdot}75$
(substituting the correct values and expressions in the correct places in the area formula) M1
$2x^2 - 3x - 27 = 0$ (convincing) A1
An attempt to solve quadratic equation in x, either by using the quadratic formula or by getting the expression into the form $(ax + b)(cx + d)$, with $a \times c = 2$ and $b \times d = -27$ M1
$(x + 3)(2x - 9) = 0 \Rightarrow x = 4{\cdot}5$ (c.a.o.) A1

(*b*) $BC^2 = 4{\cdot}5^2 + 6^2 - 2 \times 4{\cdot}5 \times 6 \times \cos 150°$
(correct use of cos rule, f.t. candidate's derived value for x) M1
$BC = 10{\cdot}15$ cm (f.t. candidate's derived value for x) A1

(*c*) $\frac{1}{2} \times 10{\cdot}15 \times AD = 6{\cdot}75$ (f.t. candidate's derived value for BC) M1
$AD = 1{\cdot}33$ cm (c.a.o.) A1

4. (*a*) $a + 14d = k \times (a + 4d)$ ($k = 7, {}^1/_7$) M1
$a + 14d = 7 \times (a + 4d)$ A1
$3a + 7d = 0$
$\frac{11}{2} \times (2a + 10d) = 88$ B1
$a + 5d = 8$
An attempt to solve the candidate's two derived linear equations simultaneously by eliminating one unknown M1
$d = 3$ (c.a.o.) A1
$a = -7$ (f.t. candidate's value for d) A1

(*b*) $-7 + (n - 1) \times 3 = 143$ (f.t. candidate's values for a and d) M1
$n = 51$ (c.a.o.) A1

5. (*a*) $S_n = a + ar + \ldots + ar^{n-1}$ (at least 3 terms, one at each end) B1

$rS_n = \quad ar + \ldots + ar^{n-1} + ar^n$

$S_n - rS_n = a - ar^n$ (multiply first line by *r* and subtract) M1

$(1 - r)S_n = a(1 - r^n)$

$S_n = \dfrac{a(1 - r^n)}{1 - r}$ (convincing) A1

(*b*) $a + ar = 25{\cdot}2$ or $\dfrac{a(1 - r^2)}{(1 - r)} = 25{\cdot}2$ B1

$\dfrac{a}{1 - r} = 30$ B1

An attempt to solve the candidate's derived equations simultaneously by eliminating *a* M1

$30(1 - r) + 30(1 - r)r = 25{\cdot}2$ (a correct quadratic in *r*) A1

$r = 0{\cdot}4$ (c.a.o.) A1

$a = 18$ (f.t. candidate's value for *r* provided $r > 0$) A1

6. (*a*) $4 \times \dfrac{x^{-2}}{-2} - 3 \times \dfrac{x^{5/4}}{5/4} + c$ (Deduct 1 mark if no *c* present) B1,B1

(*b*) (i) $4 - x^2 = 0$ M1

$x = -2,\ x = 2$ (both values, c.a.o.) A1

(ii) Use of integration to find an area M1

$\int 4\,dx = 4x, \quad \int x^2\,dx = \dfrac{x^3}{3}$ B1, B1

$$\text{Total area} = \int_{-2}^{2} (4 - x^2)\,dx - \int_{2}^{3} (4 - x^2)\,dx$$

(subtraction of integrals with correct use of candidate's x_A, x_B and 3 as limits) m1

$$\text{Total area} = \left[4x - (1/3)x^3\right]_{-2}^{2} - \left[4x - (1/3)x^3\right]_{2}^{3}$$

$$= \{[8 - (8/3)] - [(-8) - (-8/3)]\} - \{[12 - 9] - [8 - (8/3)]\}$$

Correct method of substitution of candidate's limits in at least one integral m1

Total area = 13 (c.a.o.) A1

Note: Answer only with no working shown earns 0 marks

7. (*a*) Let $p = \log_a x$, $q = \log_a y$

Then $x = a^p$, $y = a^q$ (relationship between log and power) B1

$xy = a^p \times a^q = a^{p+q}$ (the laws of indices) B1

$\log_a xy = p + q$ (relationship between log and power)

$\log_a xy = p + q = \log_a x + \log_a y$ (convincing) B1

(*b*) **Either:**

$(3 - 5x)\log_{10} 2 = \log_{10} 12$

(taking logs on both sides and using the power law) M1

$x = \dfrac{3\log_{10} 2 - \log_{10} 12}{5\log_{10} 2}$ A1

$x = -0{\cdot}117$ (f.t. one slip, see below) A1

Or:

$3 - 5x = \log_2 12$ (rewriting as a log equation) M1

$x = \dfrac{3 - \log_2 12}{5}$ A1

$x = -0{\cdot}117$ (f.t. one slip, see below) A1

Note: an answer of $x = 0{\cdot}117$ from $x = \dfrac{\log_{10} 12 - 3\log_{10} 2}{5\log_{10} 2}$

earns M1 A0 A1

an answer of $x = 1{\cdot}317$ from $x = \dfrac{\log_{10} 12 + 3\log_{10} 2}{5\log_{10} 2}$

earns M1 A0 A1

an answer of $x = -0{\cdot}585$ from $x = \dfrac{3\log_{10} 2 - \log_{10} 12}{\log_{10} 2}$

earns M1 A0 A1

Note: Answer only with no working shown earns 0 marks

(*c*) (i) $2\log_9 (x + 1) = \log_9 (x + 1)^2$ (power law) B1

$\log_9 (3x - 1) + \log_9 (x + 4) - \log_9 (x + 1)^2$

$= \log_9 \dfrac{(3x - 1)(x + 4)}{(x + 1)^2}$

(addition law) B1

(subtraction law) B1

(ii) $\log_9 \dfrac{(3x - 1)(x + 4)}{(x + 1)^2} = 1/2 \Rightarrow \dfrac{(3x - 1)(x + 4)}{(x + 1)^2} = 3$

(f.t. candidate's log expression) M1

$x = 1{\cdot}4$ (c.a.o.) A1

8. (*a*) (i) $A(4, -1)$ B1

(ii) A correct method for finding radius M1

Radius = $\sqrt{50}$ (convincing) A1

(iii) Equation of C: $(x-4)^2 + (y-[-1])^2 = 50$

(f.t. candidate's coordinates for A) B1

(*b*) **Either:**

An attempt to substitute the coordinates of R in the candidate's equation for C M1

Verification that L.H.S. of equation of $C = 50 \Rightarrow R$ lies on C A1

Or:

An attempt to find AR^2 M1

$AR^2 = 50 \Rightarrow R$ lies on C A1

(*c*) **Either:**

$RQ = \sqrt{160}$ $(RP = \sqrt{40})$ B1

$\cos PQR = \dfrac{\sqrt{160}}{2\sqrt{50}}$ $\left\{\sin PQR = \dfrac{\sqrt{40}}{2\sqrt{50}}\right\}$ M1

$PQR = 26{\cdot}565°$ (f.t. one numerical slip) A1

Or:

$RQ = \sqrt{160}$ and $RP = \sqrt{40}$ (both values) B1

$(\sqrt{40})^2 = (\sqrt{160})^2 + (2\sqrt{50})^2 - 2 \times (\sqrt{160}) \times (2\sqrt{50}) \times \cos PQR$

(correct use of cos rule) M1

$PQR = 26{\cdot}565°$ (f.t. one numerical slip) A1

9. (*a*) $\frac{1}{2} \times 5^2 \times \theta + \frac{1}{2} \times 5^2 \times \phi = 22{\cdot}5$ M1

$\theta + \phi = 1{\cdot}8$ (convincing) A1

(*b*) $5 \times \theta - 5 \times \phi = 3{\cdot}5$ or $5 \times \phi - 5 \times \theta = 3{\cdot}5$ M1

$5 \times \theta - 5 \times \phi = 3{\cdot}5$ (o.e.) A1

An attempt to solve the candidate's two linear equations simultaneously by eliminating one unknown M1

$\theta = 1{\cdot}25$, $\phi = 0{\cdot}55$ (both values) A1

(f.t. only for $\theta = 0{\cdot}55$, $\phi = 1{\cdot}25$ from $5 \times \phi - 5 \times \theta = 3{\cdot}5$)

Mathematics C3

1. (*a*)

0	1		
$\pi/12$	$0{\cdot}933012701$		
$\pi/6$	$0{\cdot}75$		
$\pi/4$	$0{\cdot}5$	(5 values correct)	B2
$\pi/3$	$0{\cdot}25$	(3 or 4 values correct)	B1

Correct formula with $h = \pi/12$ — M1

$I \approx \frac{\pi/12}{3} \times \{1 + 0{\cdot}25 + 4(0{\cdot}933012701 + 0{\cdot}5) + 2(0{\cdot}75)\}$

$I \approx 8{\cdot}482050804 \times (\pi/12) \div 3$

$I \approx 0{\cdot}740198569$

$I \approx 0{\cdot}7402$ (f.t. one slip) — A1

Note: Answer only with no working shown earns 0 marks

(*b*) $\int_0^{\pi/3} \sin^2 x \, dx = \int_0^{\pi/3} 1 \, dx - \int_0^{\pi/3} \cos^2 x \, dx$ — M1

$\int_0^{\pi/3} \sin^2 x \, dx = 0{\cdot}3070$ (f.t. candidate's answer to (*a*)) — A1

Note: Answer only with no working shown earns 0 marks

2. (*a*) e.g. $\theta = \pi/2$, $\phi = \pi$

$\sin(\theta + \phi) = -1$ (choice of θ, ϕ and one correct evaluation) — B1

$\sin\theta + \sin\phi = 1$ (both evaluations correct but different) — B1

(*b*) $\sec^2\theta + 8 = 4(\sec^2\theta - 1) + 5\sec\theta$.

(correct use of $\tan^2\theta = \sec^2\theta - 1$) — M1

An attempt to collect terms, form and solve quadratic equation in sec θ, either by using the quadratic formula or by getting the expression into the form $(a\sec\theta + b)(c\sec\theta + d)$, with $a \times c$ = candidate's coefficient of $\sec^2\theta$ and $b \times d$ = candidate's constant — m1

$3\sec^2\theta + 5\sec\theta - 12 = 0 \Rightarrow (3\sec\theta - 4)(\sec\theta + 3) = 0$

$\Rightarrow \sec\theta = \frac{4}{3}$, $\sec\theta = -3$

$\Rightarrow \cos\theta = \frac{3}{4}$, $\cos\theta = -\frac{1}{3}$ (c.a.o.) — A1

$\theta = 41{\cdot}41°, 318{\cdot}59°$

B1

$\theta = 109{\cdot}47°, 250{\cdot}53°$ — B1 B1

Note: Subtract 1 mark for each additional root in range for each branch, ignore roots outside range.
$\cos\theta$ = +, –, f.t. for 3 marks, $\cos\theta$ = –, –, f.t. for 2 marks
$\cos\theta$ = +, +, f.t. for 1 mark

3. (*a*) (i) candidate's x-derivative = $6t$,
candidate's y-derivative = $6t^5 - 12t^2$

(at least two of the three terms correct) B1

$\frac{dy}{dx} = \frac{\text{candidate's } y\text{-derivative}}{\text{candidate's } x\text{-derivative}}$ M1

$\frac{dy}{dx} = \frac{6t^5 - 12t^2}{6t}$ (c.a.o.) A1

(ii) $\frac{6t^5 - 12t^2}{6t} = \frac{7}{2}$ (f.t. candidate's expression from (i)) M1

$2t^4 - 4t - 7 = 0$

(convincing) A1

(*b*) $f(t) = 2t^4 - 4t - 7$

An attempt to check values or signs of $f(t)$ at $t = 1$, $t = 2$ M1

$f(1) = -9 < 0$, $f(2) = 17 > 0$

Change of sign $\Rightarrow f(t) = 0$ has root in (1, 2) A1

$t_0 = 1{\cdot}6$

$t_1 = 1{\cdot}608861654$ (t_1 correct, at least 5 places after the point) B1

$t_2 = 1{\cdot}609924568$

$t_3 = 1{\cdot}610051919$

$t_4 = 1{\cdot}610067175 = 1{\cdot}61007$ (t_4 correct to 5 decimal places) B1

An attempt to check values or signs of $f(t)$ at $t = 1{\cdot}610065$,
$t = 1{\cdot}610075$ M1

$f(1{\cdot}610065) = -1{\cdot}25 \times 10^{-4} < 0$, $f(1{\cdot}610075) = 1{\cdot}69 \times 10^{-4} > 0$ A1

Change of sign $\Rightarrow \alpha = 1{\cdot}61007$ correct to five decimal places A1

Note: 'Change of sign' must appear at least once.

4. $\frac{d}{dx}(x^2y^2) = x^2 \times 2y\frac{dy}{dx} + 2x \times y^2$ B1

$\frac{d}{dx}(2y^3) = 6y^2 \times \frac{dy}{dx}$ B1

$\frac{d}{dx}(x^4 - 2x + 6) = 4x^3 - 2$ B1

$x = 2, y = 3 \Rightarrow \frac{dy}{dx} = \frac{66}{30} = \frac{11}{5}$ (o.e.) (c.a.o.) B1

5. (*a*) $\frac{dy}{dx} = \frac{4}{1+(4x)^2}$ **or** $\frac{1}{1+(4x)^2}$ **or** $\frac{4}{1+4x^2}$ M1

$\frac{dy}{dx} = \frac{4}{1+16x^2}$ A1

(*b*) $\frac{dy}{dx} = e^{x^3} \times f(x)$ $(f(x) \neq 1)$ M1

$\frac{dy}{dx} = 3x^2 \times e^{x^3}$ A1

(*c*) $\frac{dy}{dx} = x^5 \times f(x) + \ln x \times g(x)$ $(f(x), g(x) \neq 1)$ M1

$\frac{dy}{dx} = x^5 \times f(x) + \ln x \times g(x)$ (either $f(x) = 1/x$ or $g(x) = 5x^4$) A1

$\frac{dy}{dx} = x^4 + 5x^4 \times \ln x$ (c.a.o.) A1

(*d*) $\frac{dy}{dx} = \frac{(5-4x^2) \times f(x) - (3-2x^2) \times g(x)}{(5-4x^2)^2}$ $(f(x), g(x) \neq 1)$ M1

$\frac{dy}{dx} = \frac{(5-4x^2) \times f(x) - (3-2x^2) \times g(x)}{(5-4x^2)^2}$

(either $f(x) = -4x$ or $g(x) = -8x$) A1

$\frac{dy}{dx} = \frac{4x}{(5-4x^2)^2}$ (c.a.o.) A1

6. (*a*) (i) $\int \sin(x/4)\,dx = k \times \cos(x/4) + c$ $(k = -1, 4, \mathbf{-4}, -\tfrac{1}{4})$ M1

$\int \sin(x/4)\,dx = -4 \times \cos(x/4) + c$ A1

(ii) $\int e^{2x/3}dx = k \times e^{2x/3} + c$ $(k = 1, \tfrac{2}{3}, \mathbf{\tfrac{3}{2}})$ M1

$\int e^{2x/3}dx = \tfrac{3}{2} \times e^{2x/3} + c$ A1

(iii) $\int \frac{7}{8x-2}\,dx = k \times 7 \times \ln|8x-2| + c$ $(k = 1, 8, \mathbf{\tfrac{1}{8}})$ M1

$\int \frac{7}{8x-2}\,dx = \tfrac{1}{8} \times 7 \times \ln|8x-2| + c$ A1

Note: The omission of the constant of integration is only penalised once.

(*b*) $\int (5x+4)^{-1/2}\,dx = k \times \frac{(5x+4)^{1/2}}{1/2}$ $(k = 1, 5, {}^{1}/_{5})$ M1

$\int_1^9 3 \times (5x+4)^{-1/2}\,dx = \left[3 \times {}^{1}/_{5} \times \frac{(5x+4)^{1/2}}{1/2}\right]_1^9$ A1

A correct method for substitution of limits in an expression of the form $m \times (5x+4)^{1/2}$ M1

$\int_1^9 3 \times (5x+4)^{-1/2}\,dx = \frac{42}{5} - \frac{18}{5} = \frac{24}{5} = 4{\cdot}8$

(f.t. only for solutions of 24 and 120 from $k = 1, 5$ respectively) A1

Note: Answer only with no working shown earns 0 marks

7. (*a*) Trying to solve either $4x - 5 \geq 3$ or $4x - 5 \leq -3$ M1

$4x - 5 \geq 3 \Rightarrow x \geq 2$

$4x - 5 \leq -3 \Rightarrow x \leq {}^{1}/_{2}$ (solving both inequalities correctly) A1

Required range: $x \leq {}^{1}/_{2}$ or $x \geq 2$ (f.t. one slip) A1

Alternative mark scheme

$(4x-5)^2 \geq 9$ (forming and trying to solve quadratic) M1

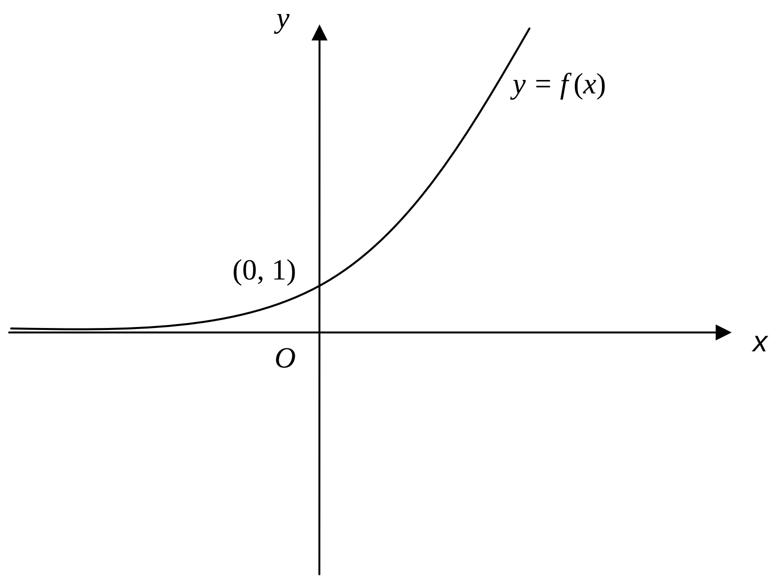
Critical values $x = {}^{1}/_{2}$ and $x = 2$ A1

Required range: $x \leq {}^{1}/_{2}$ or $x \geq 2$ (f.t. one slip) A1

(*b*) $(3|x| + 1)^{1/3} = 4 \Rightarrow 3|x| + 1 = 4^3$ M1

$x = \pm 21$ A1

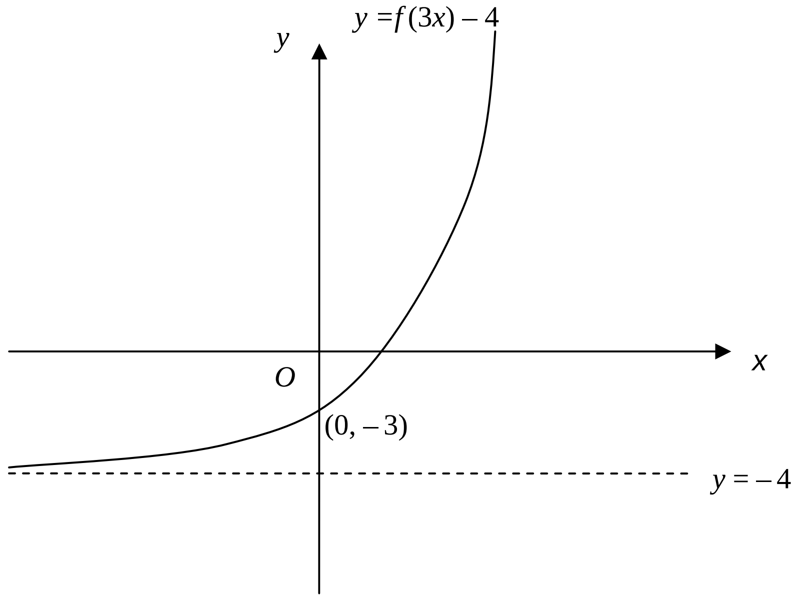
8. (*a*)



Correct shape, including the fact that the *x*-axis is an asymptote for $y = f(x)$ at $-\infty$ B1

$y = f(x)$ cuts *y*-axis at (0, 1) B1

(*b*) (i)



Correct shape, including the fact that $y = -4$ is an asymptote for $y = f(3x) - 4$ at $-\infty$ B1

(ii) $y = f(3x) - 4$ at cuts y-axis at $(0, -3)$ B1

(iii) $e^{3x} = 4 \Rightarrow 3x = \ln 4$ M1
$x = 0{\cdot}462$ A1

Note: Answer only with no working shown earns M0 A0

9. (*a*) $y = 3 - \dfrac{1}{\sqrt{x-2}} \Rightarrow 3 \pm y = \pm\dfrac{1}{\sqrt{x-2}}$ (separating variables) M1

$x - 2 = \dfrac{1}{(3 \pm y)^2}$ or $\dfrac{1}{(y \pm 3)^2}$ m1

$x = 2 + \dfrac{1}{(3-y)^2}$ (c.a.o.) A1

$f^{-1}(x) = 2 + \dfrac{1}{(3-x)^2}$ (f.t. one slip) A1

(*b*) $D(f^{-1}) = [2{\cdot}5, 3)$
[2·5 B1
3) B1

10. (*a*) $R(f) = [3 + k, \infty)$ B1

(*b*) $3 + k \geq -2$ M1

$k \geq -5$ ($\Rightarrow$ least value of k is -5)

(f.t. candidate's $R(f)$ provided it is of form $[a, \infty)$ A1

(*c*) (i) $gf(x) = (3x + k)^2 - 6$ B1

(ii) $(3 \times 2 + k)^2 - 6 = 3$

(substituting 2 for x in candidate's expression for $gf(x)$ and putting equal to 3) M1

Either $k^2 + 12k + 27 = 0$ or $(6 + k)^2 = 9$ (c.a.o.) A1

$k = -3, -9$ (f.t. candidate's quadratic in k) A1

$k = -3$ (c.a.o.) A1

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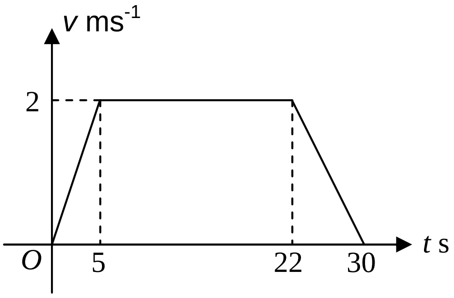
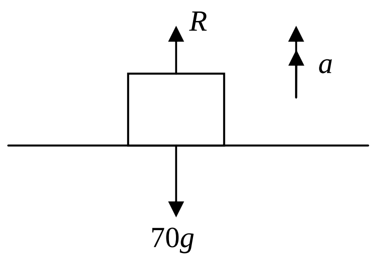
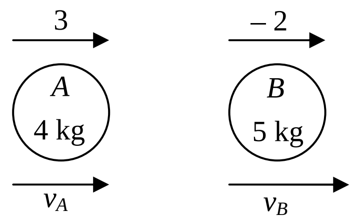
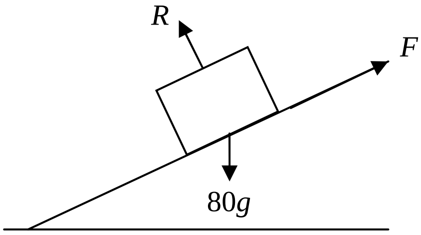
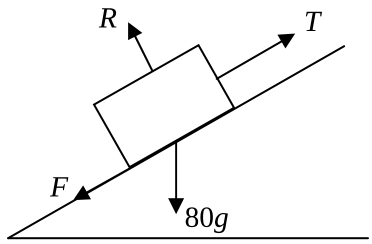
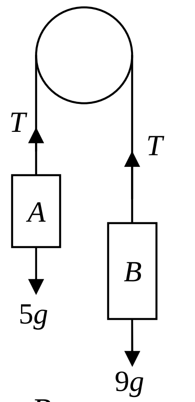
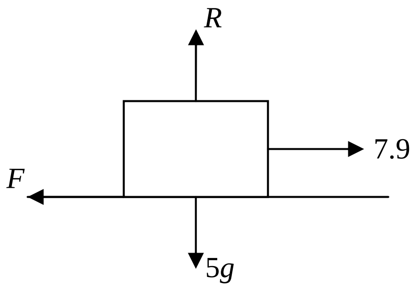
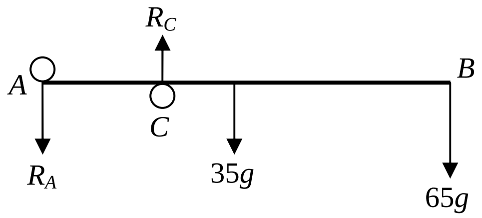
Q	Solution	Mark	Notes
1(a)	Using $v = u + at$ with $u = 0$, $a = 0.4$, $v = 2$	M1	o.e. Complete method.
	$2 = 0 + 0.4t$	A1	
	$t = \underline{5 \text{ s}}$	A1	cao
1(b)		M1	(0, 0) to (cand's t, 2)
		A1	second correct segment
		A1	all correct, labels, units
1(c)	Total distance = area under graph	M1	used, oe
	$= 0.5(17 + 30)\times 2$	A1	any expression for correct area
	$= \underline{47 \text{ m}}$	A1	cao
1(d)			
	N2L applied to man (upwards positive)	M1	R and $70g$ opposing dimensionally correct
	$R - 70g = 70a$	A1	correct equation
	Greatest R when $a = 0.4$	m1	si
	$R = 70(9.8 + 0.4)$		
	$R = \underline{714 \text{ N}}$	A1	

Q	Solution	Mark	Notes
2.			
2(a)	Conservation of momentum	M1	attempted
	$4 \times 3 + 5 \times (-2) = 4v_A + 5v_B$	A1	correct equation
	$4v_A + 5v_B = 2$		
	Restitution	M1	attempted
	$v_B - v_A == -0.2(-2 - 3)$	A1	correct equation
	$-4v_B + 4v_A = 4$	m1	attempt to eliminate
	$9v_B = 6$		
	$v_B = \frac{2}{3}$	A1	cao
	$v_A = -\frac{1}{3}$	A1	cao
2(b)	Speed after collision with wall $= 0.6v_B$	M1	
	$= 0.4$	A1	ft cand's v_B
	Impulse $= m_B\left(\frac{2}{3} + \frac{2}{5}\right)$	M1	
	$= \frac{16}{3}$ Ns	A1	ft candidate's speeds

Q	Solution	Mark	Notes
3(a)			
	Resolve perpendicular to plane	M1	dimensionally correct
	$R = 80g \cos \alpha$ $(=64g)$	A1	
3(b)	Resolve parallel to plane	M1	dimensionally correct
	$F = 80g \sin \alpha$ $(=48g)$	A1	
	$\mu = \dfrac{F}{R}$	m1	
	$\mu = \dfrac{3}{4}$	A1	cao
3(c)			
	N2L applied to body	M1	attempted. Dim correct 4 terms
	$T - F - 80g \sin \alpha = ma$	A2	-1 each error
	$F = \mu R$ $= 0.75 \times 64g$ $= 48g$ $T = 80 \times 0.7 + 48g + 48g$		
	$T = \underline{996.8 \text{ N}}$	A1	ft μ

Q	Solution	Mark	Notes
4(a)	Using $s = ut + 0.5at^2$ with $a = (\pm)9.8$, $u = 14.7$, $s = (\pm)49$ $-49 = 14.7t - 4.9t^2$ $t^2 - 3t - 10 = 0$ $(t + 2)(t - 5) = 0$ $t = \underline{5\ \text{s}}$	 M1 A1 A1	 complete method
4(b)	Using $v^2 = u^2 + 2as$ with $u = 14.7$, $a = (\pm)9.8$, $s = (\pm)49$ $v^2 = 14.7^2 + 2 \times 9.8 \times 49$ $v = \underline{34.3\ \text{ms}^{-1}}$	 M1 A1 A1	 ft t ft t
5(a)			
	Apply N2L to B $9g - T = 9a$	M1 A1	$9g$ and T opposing, dim. correct correct equ, allow –ve a
	Apply N2L to A $T - 5g = 5a$	M1 A1	$5g$ and T opposing, dim. Correct correct equ consistent With first equation
	Adding $14a = 4g$ $a = \underline{2.8\ \text{ms}^{-2}}$ $T = \underline{63\ \text{N}}$	m1 A1 A1	 cao cao
5(b)	Assuming the string to be light allows the tension throughout the string to be constant.	B1	

Q	Solution	Mark	Notes
6(a)	Resolve in 12 N direction	M1	
	$X = 12 - 16\cos 60°$	A1	
	$= 4$ N		
	Resolve in 7 N direction	M1	
	$Y = 7 - 16\cos 30°$	A1	
	Resultant = $\sqrt{(4)^2 + (-6\cdot 8565)^2}$	M1	
	= <u>7.938 N</u>	A1	cao
	$\theta = \tan^{-1}\left(\frac{6.8565}{4}\right)$	M1	allow other way up
	$\theta =$ <u>59.74°</u>	A1	ft *X, Y*
6(b)			
	$R = 5g$	B1	
	$F == 0.1R$ $(= 0.1 \times 5 \times 9.8)$	B1	ft *R*
	N2L applied to particle	M1	dim correct, all forces
	$7.9 - F = 5a$		
	$a =$ <u>0.60 ms^{-2}</u>	A1	cao

Q	Solution	Mark	Notes
7.			
7(a)	Moment of weight of rod about A		correct expression
	$= 35g \times 2$	B1	
	$= \underline{686 \text{ Nm}}$	B1	
7(b)	Take moments about A	M1	dim correct equation, all forces
	$R_C \times 1.2 = 35g \times 2 + 65g \times 4$	A1	
	$R_C = 275g$	A1	
	$= \underline{2695 \text{ N}}$		
	Resolve vertically	M1	dim correct equation, all forces
	$R_C = R_A + 35g + 65g$	A1	
	$R_A = 275g - 100g$		
	$= 175g$	A1	
	$= \underline{1715 \text{ N}}$		
8	Area from AD from AB		
	ABCD 6 1.5 1	B1	c of m of $ABCD$
	$PQRS$ 1 2 1	B1	c of m of $PQRS$
	Lamina 7 x y	B1	all areas
	$y = 1$	B1	
	Moments about AD	M1	(7 and +) or (5 and -)
	$6 \times 1.5 + 1 \times 2 = 7x$	A1	ft table
	$9 + 2 = 7x$		
	$x = \frac{11}{7}$	A1	cao

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Q	Solution	Mark	Notes
1 (a)	P(3 boys) = $\frac{6}{14}\times\frac{5}{13}\times\frac{4}{12}$ or $\binom{6}{3}\div\binom{14}{3}$	M1	
	= $\frac{5}{91}$ (0.055)	A1	
(b)	P(2 boys) = $\frac{6}{14}\times\frac{5}{13}\times\frac{8}{12}\times 3$ or $\binom{6}{2}\times\binom{8}{1}\div\binom{14}{3}$	M1A1	
	= $\frac{30}{91}$		This line need not be seen.
	P(More boys) = Sum of these probs	M1	
	= $\frac{35}{91}$ (5/13,0.385)	A1	FT previous work if first 2 M marks awarded.
2	E(Y) = 2 × 5 + 3 = 13	M1A1	M1 Use of formula, A1 answer.
	Var(X) = 5 si	B1	
	Var (Y) = 4 × 5 = 20	M1A1	M1 Use of formula, A1 answer.
3(a)(i)	P(X = 7) = $\binom{10}{7}\times 0.6^7\times 0.4^3$	M1	Accept 0.3823 – 0.1673 or 0.8327 – 0.6177 Working must be shown.
	= 0.215	A1	
(ii)	Use of the fact that if X' denote the number of times Ben wins, X' is B(10,0.4).	M1	
	We require P$(X'\le 4)$	m1	Award m1 for use of adjacent row or column.
	= 0.6331	A1	Working must be shown in (ii). Award M1 for summing probs and further 2 marks if correct.
(b)	P(1st win on 4th game) = $0.4\times 0.4\times 0.4\times 0.6$	M1A1	M1 multiplic of relevant probs.
	= 0.0384 (24/625)	A1	
4(a)	P(A∩B) = P(B) ×P(A|B)	M1	
	= 0.06	A1	
(b)	P(A∪B) = P(A) + P(B) – P(A∩B)	M1	
	= 0.54	A1	FT from (a)
(c)	P(B|A) = $\frac{P(A\cap B)}{P(A)}$	M1	
	= 0.15	A1	FT from (a) except if independence assumed.
5(a)	P(red) = $\frac{1}{3}\times\frac{1}{3}+\frac{1}{3}\times\frac{2}{3}+\frac{1}{3}\times 1$	M1A1	M1 Use of Law of Total Prob (Accept tree diagram)
	= $\frac{2}{3}$	A1	Accept Prob = No.of red cards divided by number of cards =6/9
(b)	P(A|red) = $\frac{1/9}{2/3}$	B1B1	FT denominator from (a) B1 num, B1 denom
	= $\frac{1}{6}$ cao	B1	

Q	Solution	Mark	Notes
6(a)(i)	P(X = 5) = $\dfrac{e^{-3.6} \times 3.6^5}{5!}$	M1	Working must be shown.
	= 0.138	A1	
(ii)	P(X < 3) = $e^{-3.6}\left(1 + 3.6 + \dfrac{3.6^2}{2}\right)$	M1A1	Working must be shown. Award M1 for two correct terms.
	= 0.303	A1	
(b)	$P(3 \le X \le 7) = 0.9692 - 0.3027$ or $0.6973 - 0.0308$	B1B1	B1 for each correct prob.
	= 0.666 or 0.667 (cao)	B1	
7(a)	E(X) = 0.1×1 + 0.1×2 +0.2×3 + 0.2×4 + 0.4×5	M1	M1 Use of Σxp_x.
	= 3.7	A1	
	$E(X^2)$ =0.1×1 + 0.1×4 + 0.2×9 + 0.2×16 + 0.4×25 = 15.5	B1	Need not be seen
	Var(X) = $15.5 - 3.7^2 = 1.81$	M1A1	M1 Use of correct formula for variance.
(b)	$E\left(\dfrac{1}{X^2}\right) = 0.1 \times 1 + 0.1 \times \dfrac{1}{4} + 0.2 \times \dfrac{1}{9} + 0.2 \times \dfrac{1}{16}$ $+ 0.4 \times \dfrac{1}{25}$	M1A1	M1 Use of correct formula. A1 completely correct.
	= 0.176	A1	
(c)(i)	Possibilities are 1,5; 2,4; 3,3 si	B1	
	P(Sum = 6) = 0.1×0.4×2 + 0.1×0.2×2 + 0.2×0.2	M1A1	Award M1A0 if 2s are missing
	= 0.16	A1	
(ii)	Possibilities are 1,1; 2,2; 3,3; 4,4; 5,5 si	B1	
	Prob = $0.1^2 + 0.1^2 + 0.2^2 + 0.2^2 + 0.4^2$	M1	
	= 0.26	A1	
8(a)	We are given that		
	16p(1 – p) = 2.56	M1	
	$p^2 - p + 0.16 = 0$	A1	
	Solving by a valid method	M1	
	p = 0.2 cao	A1	Award A0 if 0.2 and 0.8 given.
	Accept finding correct solution by inspection.		
(b)	$E(X^2) = \text{Var}(X) + [E(X)]^2$	M1	
	= 2.56 + 3.2^2	A1	FT on p for E(X) but not Var(X).
	= 12.8	A1	

Q	Solution	Mark	Notes
9(a)(i)	Using the fact that $F(3) = 1$, $6k = 1$ so $k = 1/6$	M1 A1	
(ii)	$P(X > 2) = 1 - F(2)$ $= 2/3$	M1 A1	
(iii)	The median satisfies $\frac{1}{6}\left(m^2 - m\right) = \frac{1}{2}$ $m^2 - m - 3 = 0$ $m = \frac{1 \pm \sqrt{1+12}}{2} = 2.30$	 M1 A1 m1A1	 M1 valid attempt to solve.
(b)(i)	$f(x) = F'(x)$ $= \frac{2x-1}{6}$	M1 A1	
(ii)	$E(X) = \frac{1}{6}\int_1^3 x(2x-1)\mathrm{d}x$ $= \frac{1}{6}\left[\frac{2x^3}{3} - \frac{x^2}{2}\right]_1^3$ $= 2.22$	M1A1 A1 A1	M1 for the integral of $xf(x)$, A1 for completely correct although limits may be left until 2nd line. FT from (b)(i) if M1 awarded there

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Q	Solution	Mark	Notes
1	$f(x+h)-f(x) = \frac{1}{(1-x-h)} - \frac{1}{(1-x)}$	M1A1	Allow division by *h* at any stage.
	$= \frac{1-x-1+x+h}{(1-x)(1-x-h)}$	A1	
	$= \frac{h}{(1-x)(1-x-h)}$	A1	
	$f'(x) = \lim_{h \to 0}\left(\frac{h}{h(1-x)(1-x-h)}\right)$	M1	
	$= \frac{1}{(1-x)^2}$ cao	A1	
2	EITHER		
	$\frac{1+3\text{i}}{1+2\text{i}} = \frac{(1+3\text{i})(1-2\text{i})}{(1+2\text{i})(1-2\text{i})}$	M1	
	$= \frac{1+3\text{i}-2\text{i}-6\text{i}^2}{1+2\text{i}-2\text{i}-4\text{i}^2}$	A1 A1	
	$= \frac{7+\text{i}}{5}$	A1	
	Modulus = $\sqrt{2}$, Argument = 8.1°, or 0.14 rad	A1A1	FT on line above.
	OR		
	Mod(1 + 3i) = $\sqrt{10}$, Arg(1 + 3i) = 71.57° or 1.249	B1B1	
	Mod(1 + 2i) = $\sqrt{5}$, Arg(1 + 2i) = 63.43° or 1.107	B1B1	
	Reqd mod = $\sqrt{2}$, Reqd arg = 8.1° or 0.14 rad	B1B1	FT on lines above.
3(a)	Let the roots be α, 2α	M1	
	Then $3\alpha = -\frac{b}{a}, 2\alpha^2 = \frac{c}{a}$	A1	
	Eliminating α,	M1	
	$\frac{b^2}{9a^2} = \frac{c}{2a}$	A1	
(b)	$ac = \frac{2b^2}{9}$		
	$b^2 - 4ac = b^2 - \frac{8}{9}b^2$	M1	Accept other valid methods
	> 0	A1	
	Therefore the roots are real.		

Q	Solution	Mark	Notes
4(a)	$(2+3i)^3 = 8+3\times4\times3i+3\times2\times(3i)^2+(3i)^3$	M1	
	$= -46 + 9i$	A1	
(b)(i)	Consider		
	$(2+3i)^3 - 3(2+3i) + 52 = -46+9i-6-9i+52$	M1	
	$= 0$	A1	
	This shows that 2 + 3i is a root of the equation.		
(ii)	Another root is 2 – 3i.	B1	
	Let the third root be α.		
	Then $2 + 3i + 2 - 3i + \alpha = 0$	M1	Accept any valid method including long division.
	$\alpha = -4$	A1	
5(a)	Determinant = $k(k-4) + 6 - 1$	M1	
	$= k^2 - 4k + 5$	A1	
	$= (k-2)^2 + 1$	M1	Allow $b^2 - 4ac = -4$
	> 0 for all real k	A1	< 0
	Therefore the matrix is non-singular for all real k		Award the second A1 only if the line above is correct.
(b)(i)	$\mathbf{A} = \begin{bmatrix} 3 & 1 & 6 \\ 1 & 3 & 4 \\ 0 & 1 & 1 \end{bmatrix}$		
	Cofactor matrix = $\begin{bmatrix} -1 & -1 & 1 \\ 5 & 3 & -3 \\ -14 & -6 & 8 \end{bmatrix}$	M1 A1	Award M1 if at least 5 cofactors are correct.
	Adjugate matrix = $\begin{bmatrix} -1 & 5 & -14 \\ -1 & 3 & -6 \\ 1 & -3 & 8 \end{bmatrix}$	A1	No FT on cofactor matrix.
(ii)	Determinant = 2	B1	FT their expression in (a)
	Inverse matrix = $\frac{1}{2}\begin{bmatrix} -1 & 5 & -14 \\ -1 & 3 & -6 \\ 1 & -3 & 8 \end{bmatrix}$	B1	FT previous work
(iii)	$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{2}\begin{bmatrix} -1 & 5 & -14 \\ -1 & 3 & -6 \\ 1 & -3 & 8 \end{bmatrix}\begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$	M1	FT their inverse matrix
	$= \begin{bmatrix} 4 \\ 1 \\ -2 \end{bmatrix}$	A1	

Q	Solution	Mark	Notes
6	Putting n = 1, the formula gives 1×2 = 1×2×3/3 which is correct so true for n = 1 Let the formula be true for $n = k$, ie $\sum_{r=1}^{k} r(r+1) = \frac{k(k+1)(k+2)}{3}$ Consider (for $n = k + 1$) $\sum_{r=1}^{k+1} r(r+1) = \sum_{r=1}^{k} r(r+1) + (k+1)(k+2)$ $= \frac{k(k+1)(k+2)}{3} + (k+1)(k+2)$ $= \frac{(k+1)(k+2)(k+3)}{3}$ Therefore true for $n = k \Rightarrow$ true for $n = k + 1$ and since true for n = 1, the result is proved by induction.	B1 M1 M1 A1 A1 A1	Award this A1 only if a correct concluding statement is made and the proof is correctly laid out
7(a)	Translation matrix = $\begin{bmatrix} 1 & 0 & h \\ 0 & 1 & k \\ 0 & 0 & 1 \end{bmatrix}$ Rotation matrix = $\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ **T** = $\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & h \\ 0 & 1 & k \\ 0 & 0 & 1 \end{bmatrix}$ $= \begin{bmatrix} 0 & 1 & k \\ -1 & 0 & -h \\ 0 & 0 & 1 \end{bmatrix}$	B1 B1 B1	
(b)(i)	$\begin{bmatrix} 0 & 1 & k \\ -1 & 0 & -h \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$ 3 + k = 1, − 1 − h = 3 $h = -4, k = -2$ (cao)	M1 A1 A1	Both correct.
(ii)	The general point on the line is (λ, 3λ + 1). The image of this point is given by $\begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 4 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \lambda \\ 3\lambda+1 \\ 1 \end{bmatrix} = \begin{bmatrix} 3\lambda-1 \\ -\lambda+4 \\ 1 \end{bmatrix}$ $x = 3\lambda - 1, y = -\lambda + 4$ Eliminating λ, $x + 3y = 11$	M1 m1 A1 M1 A1	Allow :- If $(x, y) \to (x', y')$ M1 $x' = y - 2$ A1 $y' = -x + 4$ A1 Then put $y = 3x$ +1 and eliminate x M1A1 FT their h,k

Q	Solution	Mark	Notes
8	Putting $z = x + \mathrm{i}y$, $\lvert x + \mathrm{i}(y - 1)\rvert = 2\lvert x + \mathrm{i}(y + 1)\rvert$ $x^2 + (y-1)^2 = 4[x^2 + (y+1)^2]$ $x^2 + y^2 - 2y + 1 = 4x^2 + 4y^2 + 8y + 4$ $x^2 + y^2 + \frac{10}{3}y + 1 = 0$ oe (cao) Therefore a circle. Completing the square, $x^2 + \left(y + \frac{5}{3}\right)^2 - \frac{16}{9} = 0$ Centre is $\left(0, -\frac{5}{3}\right)$, radius = $\frac{4}{3}$	**M1** **A1** **A1** **A1** **A1** **M1** **A1A1**	 FT this mark This need not be stated. FT their equation above. Allow the use of standard formulae for the radius and coordinates of the centre.
9(a) (b) (c)	Taking logs, $\ln f(x) = x \ln \sin x$ Differentiating, $\frac{f'(x)}{f(x)} = \ln \sin x + x \cot x$ $f'(x) = f(x)(\ln \sin x + x \cot x)$ oe (cao) $g(x) = \ln\sin x + x\cot x$ At a stationary point, $f'(x) = 0$ or $g(x) = 0$ $g(0.39) = -0.0183...$ or $f'(0.39) = -0.0125...$ $g(0.40) = 0.00298...$ or $f'(0.4) = 0.00204...$ The change of sign indicates that α lies between 0.39 and 0.40. Differentiating the expression in (a), $f''(x) = f'(x)g(x) + f(x)g'(x)$ Putting $x = \alpha$ and noting that $f'(\alpha)$ or $g(\alpha) = 0$, $f''(\alpha) = f(\alpha)g'(\alpha)$ $g'(x) = 2\cot x - x\mathrm{cosec}^2 x$ $g'(0.399) = 2.100...$ or $f(0.399)g'(0.399) = 1.440..$ Since this is positive, this shows that the point is a minimum.	 **B1** **B1B1** **B1** **M1** **A1** **A1** **M1** **A1** **A1** **M1A1** **A1** **A1**	 B1 for LHS, B1 for RHS This line doesn't have to be seen FT on line above. Accept any valid method for classifying the stationary point including reference to (b) above.

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