GCE AS/A level

0977/01

MATHEMATICS FP1

Further Pure Mathematics

A.M. FRIDAY, 27 January 2012

1½ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Differentiate $\dfrac{1}{1-x}$ from first principles. [6]

2. Find the modulus and the argument of the complex number

$$\frac{1+3\mathrm{i}}{1+2\mathrm{i}}.$$ [6]

3. Consider the quadratic equation $ax^2 + bx + c = 0$, where a, b, c are real.
Given that one of the roots is double the other root,

(a) show that

$$ac = \frac{2b^2}{9},$$ [4]

(b) deduce that both roots are real. [2]

4. *(a)* Express $(2 + 3\mathrm{i})^3$ in the form $x + \mathrm{i}y$, where x, y are real. [2]

(b) Hence

(i) show that $2 + 3\mathrm{i}$ is a root of the cubic equation

$$x^3 - 3x + 52 = 0,$$

(ii) find the other two roots of the equation. [5]

5. The matrix **A** is defined by

$$\mathbf{A} = \begin{bmatrix} k & 1 & 6 \\ 1 & k & 4 \\ 0 & 1 & 1 \end{bmatrix}.$$

(a) Show that **A** is non-singular for all real values of k. [4]

(b) Given that $k = 3$,

(i) find the adjugate matrix of **A**,

(ii) find the inverse matrix of **A**,

(iii) **hence** solve the equations

$$\begin{aligned} 3x + y + 6z &= 1, \\ x + 3y + 4z &= -1, \\ y + z &= -1. \end{aligned}$$ [7]

6. Use mathematical induction to prove that, for all positive integers n,

$$\sum_{r=1}^{n} r(r+1) = \frac{n(n+1)(n+2)}{3}.$$ [6]

7. The transformation T in the plane consists of a translation in which the point (x, y) is transformed to the point $(x + h, y + k)$ followed by a clockwise rotation through 90° about the origin.

(a) Show that the matrix representing T is

$$\begin{bmatrix} 0 & 1 & k \\ -1 & 0 & -h \\ 0 & 0 & 1 \end{bmatrix}.$$ [3]

(b) Given that the fixed point of T is (1, 3),

(i) find the values of h and k,

(ii) find the equation of the image of the line $y = 3x + 1$ under T. [8]

8. The complex number z is represented by the point $P(x, y)$ in the Argand diagram. Given that

$$|z - \mathrm{i}| = 2|z + \mathrm{i}|,$$

show that the locus of P is a circle and find its radius and the coordinates of its centre. [8]

9. The function f is defined, for $0 < x < 1$, by

$$f(x) = (\sin x)^x.$$

(a) Use logarithmic differentiation to show that

$$f'(x) = f(x)g(x),$$

where $g(x)$ is to be determined. [4]

(b) The graph of f has one stationary point. Show that its x-coordinate, α, lies between 0·39 and 0·40. [3]

(c) Show that

$$f''(\alpha) = f(\alpha)g'(\alpha).$$

Given that the value of α is 0·399, correct to three significant figures, determine whether the stationary point is a maximum or a minimum. [7]