GCE AS/A level

0973/01

MATHEMATICS C1
Pure Mathematics

A.M. FRIDAY, 13 January 2012

1½ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

Calculators are **not** allowed for this paper.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. The points A, B, C, D have coordinates $(-5, 14)$, $(1, 2)$, $(5, 4)$, $(3, 8)$ respectively.

(a) (i) Show that AB and CD are parallel.

(ii) Find the equation of AB.

(iii) The line L passes through the point D and is perpendicular to AB. Show that L has equation

$$x - 2y + 13 = 0.$$ [8]

(b) The lines L and AB intersect at the point E.

(i) Find the coordinates of E.

(ii) Calculate the length of EF, where F denotes the mid-point of AB. [6]

2. Simplify

(a) $\dfrac{9 + 4\sqrt{2}}{5 + 3\sqrt{2}}$, [4]

(b) $\left(\sqrt{8} \times \sqrt{10}\right) + \dfrac{\sqrt{90}}{\sqrt{2}} - \dfrac{30}{\sqrt{5}}$. [4]

3. The curve C has equation $y = 2x^2 - 8x + 13$. The point P, whose x-coordinate is 3, lies on the curve C. Find the equation of the **normal** to C at P. [6]

4. *(a)* Use the binomial theorem to expand $\left(x + \dfrac{3}{x}\right)^4$, simplifying each term of the expansion. [4]

(b) The coefficient of x^2 in the expansion of $(1 + 2x)^n$ is 760. Given that n is a positive integer, find the value of n. [3]

5. *(a)* Express $3x^2 - 6x + 5$ in the form $a(x + b)^2 + c$, where a, b and c are constants whose values are to be found. [3]

(b) **Use your answer to part *(a)*** to find the greatest value of

$$\frac{1}{3x^2 - 6x + 11}.$$ [2]

6. Given that the quadratic equation

$$(k + 6)x^2 + 4x + (k + 3) = 0$$

has no real roots, show that

$$k^2 + 9k + 14 > 0.$$

Find the range of values of k satisfying this inequality. [7]

7. *(a)* Given that $y = 8x^2 - 5x - 6$, find $\frac{dy}{dx}$ from first principles. [5]

(b) Given that $y = \frac{a}{x} + 10\sqrt{x}$ and that $\frac{dy}{dx} = 3$ when $x = 4$,

find the value of the constant a. [4]

8. *(a)* When $ax^3 - 21x - 10$ is divided by $x - 3$, the remainder is 35.
Write down an equation satisfied by a and hence show that $a = 4$. [2]

(b) Factorise $4x^3 - 21x - 10$. [5]

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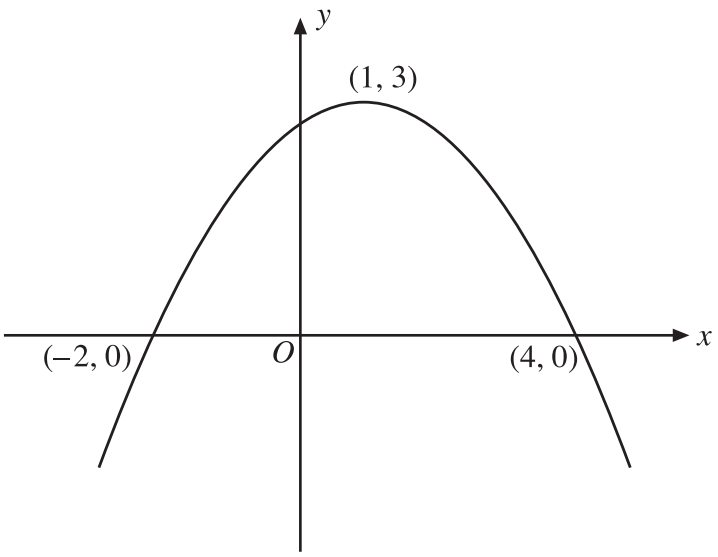
9. The diagram shows a sketch of the graph of $y = f(x)$. The graph has a maximum point at (1, 3) and intersects the x-axis at the points (–2, 0) and (4, 0).



(a) Sketch the graph of $y = f(2x)$, indicating the coordinates of the stationary point and the coordinates of the points of intersection of the graph with the x-axis. [3]

(b) (i) Sketch the graph of $y = f(x) – 5$, indicating the coordinates of the stationary point.

(ii) Given that f is a quadratic function, use the graph you have drawn in part (i) to write down the number of real roots of the equation

$$f(x) – 5 = 0.$$ [3]

10. The curve C has equation

$$y = x^3 – 6x^2 + 12x – 9.$$

(a) Show that C has only one stationary point. Find the coordinates of this point. [4]

(b) Verify that this stationary point is a point of inflection. [2]