GCE AS/A level

0976/01

MATHEMATICS C4

Pure Mathematics

A.M. THURSDAY, 14 June 2012

1½ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. The function f is defined by

$$f(x) = \frac{11 + x - x^2}{(x+1)(x-2)^2}.$$

(a) Express $f(x)$ in terms of partial fractions. [4]

(b) Use your result to part *(a)* to find the value of $f'(0)$. [3]

2. Find the equation of the tangent to the curve

$$y^3 - 4x^2 - 3xy + 25 = 0$$

at the point $(2, -3)$. [4]

3. *(a)* Find all values of θ in the range $0° \leqslant \theta \leqslant 360°$ satisfying

$$4\cos 2\theta = 1 - 2\sin\theta.$$ [6]

(b) (i) Express $8\sin x + 15\cos x$ in the form $R\sin(x + \alpha)$, where R and α are constants with $R > 0$ and $0° < \alpha < 90°$.

(ii) Find all values of x in the range $0° \leqslant x \leqslant 360°$ satisfying

$$8\sin x + 15\cos x = 11.$$

(iii) Find the greatest possible value for k so that

$$8\sin x + 15\cos x = k$$

has solutions. Give a reason for your answer. [7]

4. The region R is bounded by the curve $y = \sqrt{x} + \dfrac{5}{\sqrt{x}}$, the x-axis and the lines $x = 3$, $x = 4$.

Find the volume generated when R is rotated through four right-angles about the x-axis. Give your answer correct to the nearest integer. [5]

5. Expand $\left(1 + \dfrac{x}{3}\right)^{-\frac{1}{2}}$ in ascending powers of x up to and including the term in x^2.

State the range of values of x for which your expansion is valid.

Hence, by writing $x = \dfrac{1}{5}$ in your expansion, find an approximate value for $\sqrt{15}$ in the form $\dfrac{a}{b}$, where a and b are integers whose values are to be found. [5]

6. The parametric equations of the curve C are $x = t^2$, $y = 2t$.

(a) Show that the normal to C at the point P with parameter p has equation

$$y + px = p^3 + 2p.$$ [5]

(b) The normal to C at the point P intersects C again at the point with parameter 3.

(i) Show that $p^3 - 7p - 6 = 0$.

(ii) Hence show that P can be one of two points. Find the coordinates of each of these two points. [6]

7. *(a)* Find $\int x e^{-2x}\,dx$. [4]

(b) Use the substitution $u = 1 + 3\ln x$ to evaluate

$$\int_1^e \frac{1}{x(1 + 3\ln x)}\,dx.$$

Give your answer correct to four decimal places. [4]

8. Water is leaking from a hole at the bottom of a large tank. The volume of the water in the tank at time t hours is $V\,m^3$. The rate of decrease of V is directly proportional to V^3.

(a) Write down a differential equation satisfied by V. [1]

(b) Given that $V = 60$ when $t = 0$, show that

$$V^2 = \frac{3600}{at + 1},$$

where a is a constant. [4]

(c) When $t = 2$, the volume of the water in the tank is $50\,m^3$. Find the value of t when the volume of the water in the tank is $27\,m^3$. Give your answer correct to one decimal place. [4]

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9. The position vectors of the points A and B are given by

$$\mathbf{a} = 4\mathbf{i} + \mathbf{j} - 6\mathbf{k},$$
$$\mathbf{b} = 6\mathbf{i} + 2\mathbf{j} - 4\mathbf{k},$$

respectively.

(a) Determine whether or not the vectors **a** and **b** are perpendicular, giving a reason for your answer. [2]

(b) (i) Write down the vector **AB**.
(ii) Find the vector equation of the line AB. [3]

(c) The vector equation of the line L is given by

$$\mathbf{r} = 2\mathbf{i} + 6\mathbf{j} + p\mathbf{k} + \mu(-2\mathbf{i} + \mathbf{j} + 3\mathbf{k}),$$

where p is a constant.
Given that the lines AB and L intersect, find the value of p. [5]

10. Complete the following proof by contradiction to show that $\sqrt{5}$ is irrational.

Assume that $\sqrt{5}$ is rational. Then $\sqrt{5}$ may be written in the form $\frac{a}{b}$, where a, b are integers having no common factors.

$\therefore$ *$a^2 = 5b^2$.*
$\therefore$ *a^2 has a factor 5.*
$\therefore$ *a has a factor 5 so that $a = 5k$, where k is an integer.* [3]