GCE AS/A level

977/01

MATHEMATICS FP1

Further Pure Mathematics

P.M. MONDAY, 31 January 2011

$1\frac{1}{2}$ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Given that

$$S_n = (1^2 \times 3) + (2^2 \times 5) + (3^2 \times 7) + \ldots + n^2\,(2n + 1),$$

obtain an expression for S_n in terms of n, giving your answer as a product of two linear factors and a quadratic factor. [5]

2. Consider the following equations.

$$\begin{aligned} x + 2y + z &= 1 \\ 2x + 3y + z &= 3 \\ 3x + 4y + z &= \lambda \end{aligned}$$

Given that these equations are consistent,

(a) find the value of λ, [4]

(b) find the general solution. [3]

3. The complex number z satisfies the equation

$$\frac{1}{z} - 4(1 - \mathrm{i}) = (2 + \mathrm{i})(-1 + \mathrm{i})$$

(a) Find z in the form $x + \mathrm{i}y$. [6]

(b) Find the modulus and argument of z. [2]

4. The roots of the cubic equation

$$x^3 - 3x^2 + 2x + 4 = 0$$

are denoted by α, β, γ.

(a) Show that

$$\frac{\beta\gamma}{\alpha} + \frac{\gamma\alpha}{\beta} + \frac{\alpha\beta}{\gamma} = -7.$$ [5]

(b) Find the cubic equation whose roots are $\frac{\beta\gamma}{\alpha}, \frac{\gamma\alpha}{\beta}, \frac{\alpha\beta}{\gamma}$. [6]

5. Use mathematical induction to prove that

$$\begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix}^n = \begin{bmatrix} 1 & 2^n - 1 \\ 0 & 2^n \end{bmatrix}$$

for all positive integers n. [7]

6. The matrix **A** is given by

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 3 \\ \lambda & 1 & -2 \\ 2 & 1 & \lambda \end{bmatrix}.$$

(a) (i) Find and simplify an expression for the determinant of **A**.

(ii) Show that **A** is non-singular for all real values of λ. [4]

(b) Given that $\lambda = 1$,

(i) find $\mathbf{A}^{-1}$, the inverse of **A**,

(ii) hence solve the equation $\mathbf{AX} = \mathbf{B}$,

$$\text{where } \mathbf{X} = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \quad \text{and } \mathbf{B} = \begin{bmatrix} 9 \\ 2 \\ 7 \end{bmatrix}.$$ [7]

7. The function f is defined for $x > 0$ by

$$f(x) = 2^x \times 3^{\frac{1}{x}}.$$

(a) Use logarithmic differentiation to obtain an expression for $f'(x)$ in terms of x. [4]

(b) Find the stationary value of $f(x)$ and determine whether it is a maximum or a minimum. [4]

8. The transformation T in the plane consists of a reflection in the line $y - x = 0$, followed by a translation in which the point (x, y) is transformed to the point $(x + 2, y - 1)$, followed by a reflection in the line $y + x = 0$.

(a) Show that the matrix representing T is

$$\begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & -2 \\ 0 & 0 & 1 \end{bmatrix}.$$ [5]

(b) Find the coordinates of the fixed point of T. [3]

9. The complex numbers z and w are represented, respectively, by points $P(x, y)$ and $Q(u, v)$ in Argand diagrams and $w = z^2$.

(a) Obtain expressions for u and v in terms of x and y. [3]

(b) The point P moves along the curve with equation $y = x^2$. Find the equation of the locus of Q, giving your answer in the form $u = f(v)$. [3]

(c) The point $R(\alpha, 16)$ lies on the locus of Q.

(i) Find the value of α.

(ii) Find the coordinates of the point on the locus of P which corresponds to R. [4]