GCE AS/A level

975/01

MATHEMATICS C3

Pure Mathematics

P.M. WEDNESDAY, 19 January 2011

1½ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Use Simpson's Rule with five ordinates to find an approximate value for the integral

$$\int_4^6 \frac{1}{3-\sqrt{x}}\,\mathrm{d}x.$$

Show your working and give your answer correct to three decimal places. [4]

2. *(a)* Show, by counter-example, that the statement

$$\sec^2\theta \equiv 1 - \operatorname{cosec}^2\theta$$

is false. [2]

(b) Find all values of θ in the range $0° \leqslant \theta \leqslant 360°$ satisfying

$$3\operatorname{cosec}^2\theta = 11 - 2\cot\theta\,.$$ [6]

3. *(a)* Given that

$$x^4 + 3x^2y - 2y^2 = 15,$$

find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x and y. [4]

(b) Given that $x = \ln t$, $y = t^3 - 7t$,

(i) find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of t,

(ii) find the value of $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ when $t = \frac{1}{3}$. [8]

4. **You may assume** that the equation $6x^4 + 7x - 3 = 0$ has a root α between 0 and 1.
The recurrence relation

$$x_{n+1} = \frac{3 - 6x_n^4}{7}$$

with $x_0 = 0{\cdot}4$ can be used to find α. Find and record the values of x_1, x_2, x_3, x_4.
Write down the value of x_4 correct to four decimal places and show this is the value of α correct to four decimal places. [5]

5. *(a)* Differentiate each of the following with respect to x, simplifying your answer wherever possible.

(i) $\sqrt{2+5x^3}$ (ii) $x^2\sin 3x$ (iii) $\frac{\mathrm{e}^{2x}}{x^4}$ [8]

(b) By first writing $y = \tan^{-1}x$ as $x = \tan y$, find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x. [4]

6. *(a)* Find

(i) $\int \cos 4x \, dx$, (ii) $\int 5e^{2-3x} \, dx$, (iii) $\int \frac{3}{(6x-7)^5} \, dx$. [6]

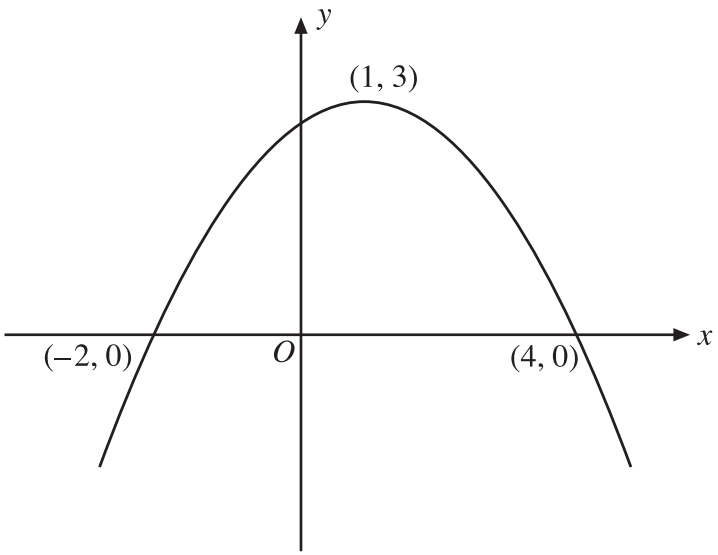
(b) Evaluate $\int_1^4 \frac{9}{2x+5} \, dx$, giving your answer correct to three decimal places. [4]

7. Solve the following.

(a) $5|x| + 1 = 7 - 3|x|$ [2]

(b) $|3x - 1| > 5$ [3]

8. The diagram shows a sketch of the graph of $y = f(x)$. The graph passes through the points $(-2, 0)$ and $(4, 0)$ and has a maximum point at $(1, 3)$.



Sketch the graph of $y = -3f(x + 2)$, indicating the coordinates of the stationary point and the coordinates of the points of intersection of the graph with the x-axis. [3]

TURN OVER

9. The function f has domain $(-\infty, -1]$ and is defined by

$$f(x) = 4x^2 - 3.$$

(a) Write down the range of f. [1]

(b) Find an expression for $f^{-1}(x)$ and write down the range and domain of f^{-1}. [5]

(c) (i) Evaluate $f^{-1}(6)$.

(ii) By carrying out an appropriate calculation involving f, verify that your answer to part (i) is correct. [3]

10. The functions f and g have domains $[0, \infty)$ and $(-\infty, \infty)$ respectively and are defined by

$$f(x) = e^x,$$
$$g(x) = 4x^3 + 7.$$

(a) Find and simplify an expression for $gf(x)$. [2]

(b) Find the domain and range of gf. [2]

(c) (i) Solve the equation $gf(x) = 18$. Give your answer correct to three decimal places.

(ii) Giving a reason, write down a value for k so that $gf(x) = k$ has no solution. [3]