WJEC
CBAC

GCE AS/A level

979/01

MATHEMATICS FP3
Further Pure Mathematics

P.M. FRIDAY, 24 June 2011

$1\frac{1}{2}$ hours

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Use black ink or black ball-point pen.

Answer **all** questions.

Sufficient working must be shown to demonstrate the **mathematical** method employed.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Find the positive root of the equation

$$3\tanh^2\theta = 5\operatorname{sech}\theta + 1,$$

giving your answer in the form $\ln(a+\sqrt{b}\,)$, where a, b are positive integers. [8]

2. Use the substitution $t = \tan\dfrac{x}{2}$ to show that

$$\int_0^{\frac{\pi}{2}} \frac{1}{2+\sin x}\,dx = \frac{\pi}{3\sqrt{3}}.$$ [8]

3. Show that the length of the arc joining the points $(2a, 2a)$ and $(4a, 2\sqrt{3}a)$ on the curve with equation $y^2 = 4a(x-a)$ is given by the integral

$$\int_{2a}^{4a} \sqrt{\frac{x}{x-a}}\,dx.$$

Hence evaluate this length using the substitution $x = a\cosh^2 u$. Give your answer in the form ka where k should be evaluated correct to three significant figures. [14]

4. The function f is defined by

$$f(x) = e^x \cos x.$$

(a) Show that

$$f''(x) = -2e^x \sin x.$$ [2]

(b) Determine the Maclaurin series for $f(x)$ as far as the term in x^4. [5]

(c) By differentiating your series, determine the Maclaurin series for $e^x \sin x$ as far as the term in x^3. [4]

5. Consider the equation $x\sin x - 0{\cdot}5 = 0$.

(a) Show that this equation has a root α between 0·6 and 0·8. [2]

(b) (i) Show that the Newton-Raphson iteration to find the value of α can be written in the form

$$x_{n+1} = \frac{x_n^2\cos x_n + 0{\cdot}5}{x_n\cos x_n + \sin x_n}\ .$$

(ii) Starting with $x_0 = 0{\cdot}7$, find the value of α correct to five decimal places. [6]

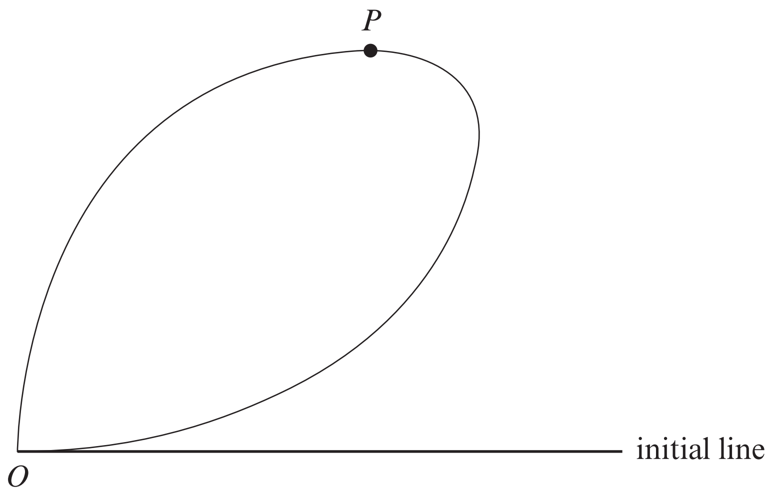
(c) A rearrangement of the equation leads to the iterative sequence

$$x_{n+1} = f(x_n) \text{ where } f(x) = \sin^{-1}\left(\frac{0{\cdot}5}{x}\right)\ .$$

(i) Obtain an expression for $f'(x)$.

(ii) **Hence** determine whether or not the sequence can be used to find the value of α. [4]

6.



The above diagram shows a sketch of the curve C with polar equation

$$r = \sin 2\theta, \quad 0 \leqslant \theta \leqslant \frac{\pi}{2}.$$

The point P, marked on the diagram, is the point at which the tangent to C is parallel to the initial line.

(a) Determine the area of the region enclosed by C. [5]

(b) Find the polar coordinates of the point P. [7]

TURN OVER

7. The integral I_n is defined, for $n \geqslant 0$, by

$$I_n = \int_0^a \tanh^n x \,\mathrm{d}x,$$

where $a = \tanh^{-1} 0{\cdot}5$.

(a) Show that, for $n \geqslant 2$,

$$I_n = I_{n-2} - \frac{0{\cdot}5^{n-1}}{n-1}.$$ [5]

(b) Giving your answers correct to three significant figures, evaluate

(i) I_0,

(ii) I_4. [5]