

981/01

**MATHEMATICS M2**

**Mechanics 2**

A.M. THURSDAY, 7 June 2007

(1½ hours)

**ADDITIONAL MATERIALS**

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

**INSTRUCTIONS TO CANDIDATES**

Answer **all** questions.

Take  $g$  as  $9.8 \text{ ms}^{-2}$ .

**INFORMATION FOR CANDIDATES**

The number of marks is given in brackets at the end of each question or part-question.

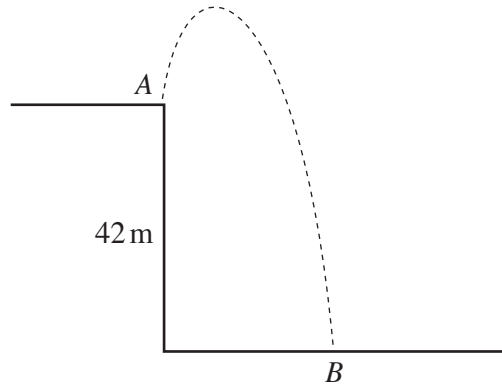
You are reminded of the necessity for good English and orderly presentation in your answers.

1. A particle  $P$  is projected from the origin  $O$  so that it moves along the  $x$ -axis. At time  $t$  s after projection, the velocity of the particle,  $v \text{ ms}^{-1}$ , is given by

$$v = 3t^2 - 24t + 45.$$

- (a) Show that  $P$  first comes to instantaneous rest when  $t = 3$ . [2]
- (b) Find an expression for the acceleration of  $P$  at time  $t$  s. [2]
- (c) Find an expression for the displacement of  $P$  from  $O$  at time  $t$  s. [3]
- (d) Find the distance travelled by the particle in the first 3 seconds of its motion. [2]
- (e) Find the distance travelled by the particle in the first 4 seconds of its motion. [2]
2. A car of mass 900 kg can produce a maximum power of 45 kW. The car experiences a constant resistance to motion of magnitude 1800 N.
- (a) Calculate the maximum speed of the car when travelling on a horizontal road. [3]
- (b) The car travels up a slope inclined at an angle of  $4^\circ$  to the horizontal. Assuming maximum power is employed, calculate, correct to two decimal places, the acceleration of the car at the instant when its speed is  $15 \text{ ms}^{-1}$ . [5]
- (c) The car travels a distance of 800 m. Calculate the work done against resistance. [2]
3. The end  $A$  of a light elastic string  $AB$ , of natural length 0.8 m, is fixed. A particle  $P$ , of mass 3 kg, is attached to the end  $B$  of the string. Initially,  $P$  is held at rest at the point  $A$ . It is then released and allowed to fall. The greatest extension of the string in the subsequent motion is 0.4 m.
- (a) Show that the modulus of elasticity of the string is 352.8 N. [7]
- (b) Find the tension in the string when  $P$  is at its lowest point and deduce the magnitude of the acceleration of  $P$  in this position. [5]

4. A stone is projected from point  $A$  on the top of a vertical cliff and it hits the sea at point  $B$ . The height of  $A$  above sea level is 42 m.



The horizontal and vertical components of the stone's initial velocity are  $4.5 \text{ ms}^{-1}$  and  $22.4 \text{ ms}^{-1}$  respectively.

- (a) Find the speed of the stone 2 s after projection. [5]
- (b) Calculate the time of flight of the stone. [4]
- (c) Determine the distance of  $B$  from the foot of the cliff. [2]

5. Vectors  $\mathbf{a}$  and  $\mathbf{b}$  are given by

$$\mathbf{a} = 2\mathbf{i} + 13\mathbf{j} - 10\mathbf{k},$$

$$\mathbf{b} = -\mathbf{i} + y\mathbf{j} + 5\mathbf{k}.$$

- (a) If  $\mathbf{a}$  and  $\mathbf{b}$  are perpendicular, find the value of  $y$ . [4]
- (b) If  $\mathbf{a}$  and  $\mathbf{b}$  are parallel, find the value of  $y$ . [2]
6. A particle of mass  $0.8 \text{ kg}$  is attached to one end of a light inextensible string of length  $0.4 \text{ m}$ . The other end of the string is fixed to a point  $O$  of a smooth horizontal surface. The particle moves on the surface with constant speed  $3 \text{ ms}^{-1}$  in a horizontal circle with centre  $O$ .
- (a) Find the angular velocity about  $O$  of the particle. [2]
- (b) Calculate the tension in the string. [2]

**TURN OVER**

7. A particle, of mass 3 kg, is attached to one end of a light rod of length 0.9 m. The other end of the rod is freely pivoted at a fixed point  $O$ . The particle moves in a vertical circle with centre  $O$ , such that its speed at the lowest point of its path is three times its speed at the highest point of its path.
- (a) Show that the speed of the particle at the lowest point of its path is  $6.3 \text{ ms}^{-1}$ . [5]
- (b) Calculate the thrust in the rod when the particle is at the highest point of its path. [4]
- (c) If a string replaced the rod, state, with a reason, whether the particle would still move in complete circles. [2]
8. A toy plane  $A$  is moving with constant velocity  $(3\mathbf{i} - 2\mathbf{j} + 5\mathbf{k}) \text{ ms}^{-1}$  and at time  $t = 0$ , its position vector is  $(3\mathbf{j} - 140\mathbf{k}) \text{ m}$ . Another toy plane  $B$  is moving with constant velocity  $(-2\mathbf{i} + 6\mathbf{j} + 3\mathbf{k}) \text{ ms}^{-1}$  and at time  $t = 0$ , its position vector is  $(-9\mathbf{i} - 4\mathbf{j} - 6\mathbf{k}) \text{ m}$ .
- (a) Write down the position vectors of  $A$  and  $B$  at time  $t$  s. [3]
- (b) Find an expression for the square of the distance between  $A$  and  $B$  at time  $t$  s. [3]
- (c) Determine the time when  $A$  and  $B$  are closest together. [4]