

976/01

MATHEMATICS C4

Pure Mathematics

P.M. THURSDAY, 14 June 2007

(1½ hours)

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. (a) Express $\frac{x+3}{x^2(x-1)}$ in terms of partial fractions. [4]

(b) Find $\int \frac{x+3}{x^2(x-1)} dx$. [2]

2. Find the equation of the tangent to the curve

$$x^5 + xy^2 + y^3 = 17$$

at the point $(-1, 3)$. [4]

3. Find all values of x in the range $0^\circ \leq x \leq 360^\circ$ satisfying the equation $4\cos x + 2\sin x = 3$. [7]

4. Expand $(1+4x)^{\frac{1}{2}} - \frac{1}{1+3x}$ as far as the term in x^2 . For what range of values of x is your expansion valid? [7]

5. The region bounded by the curve $y = \sqrt{e^{2x} + 1}$, the x -axis and the lines $x = 0, x = 1$ is rotated through four right-angles about the x -axis. Find the volume of the solid generated, giving your answer correct to three decimal places. [4]

6. The parametric equations of the curve C are $x = 2t, y = t^2$.

(a) Show that the normal to C at the point P with parameter p has equation

$$x + py = p^3 + 2p. \quad [4]$$

(b) The normal to C at the point P intersects the x -axis at A and the y -axis at B . Given that O is the origin and $OA = 2OB$, find the value of p . [4]

7. (a) Find $\int x^2 \ln x dx$. [4]

(b) Use the substitution $x = 2\sin\theta$ to show that

$$\int_0^{\sqrt{2}} \frac{x^2}{\sqrt{4-x^2}} dx = \int_0^a k \sin^2 \theta d\theta,$$

where the values of a and k are to be determined.

Hence, or otherwise, evaluate $\int_0^{\sqrt{2}} \frac{x^2}{\sqrt{4-x^2}} dx$. [8]

8. The price $\pounds P$ of an item at time t years is to be modelled as a continuous variable such that the rate of increase of P is directly proportional to P .
- (a) Write down a differential equation that is satisfied by P . [1]
- (b) Given that the price of the item at $t = 0$ is $\pounds 50$, show that $P = 50e^{kt}$, where k is a positive constant. [5]
- (c) After seven years the price of the item is $\pounds 65$. Find the price of the item after sixteen years. [4]
9. (a) The position vectors of the points A and B , relative to a fixed origin O , are $\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$ and $3\mathbf{i} + 6\mathbf{j} + \mathbf{k}$, respectively.
- (i) Find \mathbf{AB} .
- (ii) Find the vector equation of the line AB .
- (iii) The vector equation of the line L is $\mathbf{r} = 2\mathbf{i} + 3\mathbf{j} + 7\mathbf{k} + \mu(\mathbf{i} + \mathbf{j} + 4\mathbf{k})$.
Given that L and AB intersect, find the position vector of the point of intersection. [9]
- (b) Find the angle between the vectors $\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ and $3\mathbf{i} - \mathbf{j} + 2\mathbf{k}$. [6]
10. Complete the following proof by contradiction to show that, if n is a positive integer and $3n + 2n^3$ is odd, then n is odd. [2]
- It is given that $3n + 2n^3$ is odd.*
Assume that n is even so that $n = 2k$.