

983/01

MATHEMATICS S1

Statistics

P.M. THURSDAY, 18 January 2007

(1½ hours)

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator;
- statistical tables (Murdoch and Barnes or RND/WJEC Publications)

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. A bag contains 9 balls of which 2 are red, 3 are blue and 4 are yellow. Bill chooses 3 of these balls at random without replacement. Find the probability that
- (a) he chooses 3 yellow balls, [2]
 - (b) he chooses no blue balls, [2]
 - (c) he chooses 1 ball of each colour. [3]

2. The events A and B are such that

$$P(A) = 0.48, P(B) = 0.38, P(A \cap B) = 0.28.$$

Calculate

- (a) $P(A \cup B)$, [2]
 - (b) $P(A' \cap B')$, [2]
 - (c) $P(B | A')$. [4]
3. The random variable X has the distribution $B(n, 0.1)$. Given that the mean and standard deviation of X are equal, find the value of n . [5]
4. Mair chooses a number at random from the set $\{2, 3, 4\}$. Whichever number Mair chooses, she then tosses that number of fair coins.
- (a) Find the probability that all the coins tossed land 'heads'. [4]
 - (b) Given that all the coins land 'heads', find the probability that she chose the number 2. [3]
5. (a) It is known that 35% of a certain type of seed produce red flowers. A gardener buys 20 of these seeds. Find the probability that
- (i) exactly 5 seeds produce red flowers,
 - (ii) fewer than 8 seeds produce red flowers. [5]
- (b) It is also known that 3% of these seeds produce yellow flowers. The manager of a Garden Centre buys 500 of these seeds. Use a distributional approximation to find the probability that
- (i) exactly 10 seeds produce yellow flowers,
 - (ii) more than 12 seeds produce yellow flowers. [5]

6. The probability distribution of the discrete random variable X is given in the following table.

x	1	2	3	4	5
$P(X = x)$	0.3	p	0.1	q	0.05

- (a) Show that $p + q = 0.55$. [1]
- (b) Given that $E(X) = 2.75$, show that $p = 0.15$ and $q = 0.4$. [4]
- (c) Find the variance of X . [3]
- (d) The random variable Y is defined by $Y = 4X + 2$.
- (i) Find the mean and variance of Y .
- (ii) Find $P(Y < 15)$. [6]

7. The continuous random variable X has probability density function f given by

$$f(x) = 20(x^3 - x^4), \quad \text{for } 0 \leq x \leq 1,$$

$$f(x) = 0, \quad \text{otherwise,}$$

- (a) Find $E(X)$. [4]
- (b) (i) Obtain an expression for $F(x)$, valid for $0 \leq x \leq 1$, where F denotes the cumulative distribution function of X .
- (ii) Evaluate $P(0.4 \leq X \leq 0.6)$.
- (iii) The upper quartile of X is denoted by q . Show that

$$16q^5 - 20q^4 + 3 = 0. \quad [8]$$

8. Students on a typing course are each given a page to type. You may assume that the number of errors made on a page follows a Poisson distribution with mean μ where the value of μ varies from student to student.

- (a) For Alan, $\mu = 3.75$. Without the use of tables, find the probability that he makes exactly 3 errors. [2]
- (b) A page is unsatisfactory if it contains at least 5 errors.
- (i) For Belle, $\mu = 2.4$. Find the probability that her page is unsatisfactory.
- (ii) The probability that Ceri's page is unsatisfactory is 0.2194. Using tables, find the value of μ for Ceri. [4]
- (c) Diane is the best pupil in the class and for her, $\mu = 0.6$. She is given n pages to type. Assuming that each page is independent of all others,
- (i) show that the probability that there are no errors on any of these pages is $e^{-0.6n}$,
- (ii) find the minimum value of n for which this probability is less than 0.01. [6]