

977/01

MATHEMATICS FP1

Further Pure Mathematics

P.M. TUESDAY, 23 January 2007

(1½ hours)

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Find an expression, in terms of n , for the sum of the series

$$1.2.3 + 2.3.5 + 3.4.7 + \dots + n(n+1)(2n+1).$$

Express your answer as a product of linear factors.

[5]

2. (a) Find the inverse of the following matrix.

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 3 & 4 & 2 \end{bmatrix}$$

[6]

- (b) **Hence** solve the equations

$$\begin{aligned} x + 2y + z &= 1 \\ 2x + 3y + z &= 4 \\ 3x + 4y + 2z &= 4. \end{aligned}$$

[2]

3. (a) Showing your working, simplify the expression

$$\frac{(3+4i)(1+2i)}{1+3i}$$

giving your answer in the form $x + iy$.

[4]

- (b) Write down, in terms of $\arg(z_1)$ and $\arg(z_2)$,

(i) $\arg(z_1 z_2)$,

(ii) $\arg\left(\frac{z_1}{z_2}\right)$.

[1]

- (c) Use the results in (a) and (b) to show that

$$\tan^{-1}\left(\frac{4}{3}\right) + \tan^{-1} 2 - \tan^{-1} 3 = \frac{\pi}{k}$$

where k is a positive integer whose value is to be determined.

[2]

4. Use mathematical induction to show that $6^n + 4$ is divisible by 5 for all positive integers n .

[7]

5. Consider the simultaneous equations

$$\begin{aligned}x + 2y - z &= 2 \\ 2x - y + z &= 3 \\ 4x - 7y + 5z &= 5.\end{aligned}$$

Given that these equations do not have a unique solution,

- (a) show that the equations are consistent. [4]
 (b) find the general solution to the equations. [3]

6. The function f is defined on the domain $(0, \infty)$ by

$$f(x) = x^{-1 \ln x}$$

- (a) Find the coordinates of the stationary point on the graph of f . [5]
 (b) Determine the nature of this stationary point. [2]

7. The roots of the cubic equation

$$x^3 + 2x^2 + 3x - 4 = 0$$

are denoted by α, β, γ . Find the cubic equation whose roots are $\frac{\beta\gamma}{\alpha}, \frac{\gamma\alpha}{\beta}, \frac{\alpha\beta}{\gamma}$. [11]

8. The transformation T in the plane consists of an anticlockwise rotation about the origin through an angle θ followed by a translation in which the point (x, y) is transformed to the point $(x + h, y + k)$.

- (a) Find the 3×3 matrix corresponding to T . [4]
 (b) Given that T maps the point $(0, 1)$ to $(1, 2)$ and the point $(3, 0)$ to $(4, 3)$, find the values of h , k and θ . [7]

9. The complex number z is represented by the point $P(x, y)$ in an Argand diagram.

- (a) Given that

$$|z - 3| = |z + i|,$$

find the Cartesian equation of the locus of P . [5]

- (b) Find the coordinates of the two points lying on this locus for which $|z| = 4$. [7]