

974/01

MATHEMATICS C2

Pure Mathematics

P.M. WEDNESDAY, 10 January 2007

(1½ hours)

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. Use the Trapezium Rule with five ordinates to find an approximate value for

$$\int_1^2 \sqrt{2+x^3} \, dx.$$

Show your working and give your answer correct to three decimal places. [4]

2. (a) Find the values of x in the range $0^\circ \leq x \leq 360^\circ$ satisfying

$$10 \sin^2 x - 3 \sin x = 4 \cos^2 x + 1. \quad [6]$$

- (b) Find the values of x in the range $0^\circ \leq x \leq 180^\circ$ satisfying

$$\tan(2x + 30^\circ) = \sqrt{3}. \quad [3]$$

3. (a) A geometric series has first term a and common ratio r . Write down the n th term and prove that the sum of the first n terms is given by

$$S_n = \frac{a(1-r^n)}{1-r}.$$

Given that $|r| < 1$, write down the sum to infinity of the series. [5]

- (b) The sum of the first term and the second term of a geometric series is equal to twice the sum of the second term and the third term of the series.

(i) Given that the common ratio of the series is positive, find the value of the common ratio. [4]

(ii) The sum to infinity of the series is 12. Find, correct to two decimal places, the sum of the first eight terms of the series. [4]

4. In an arithmetic series, the eighth term is twice the third term. The twentieth term of the series is 11. Find the common difference and the first term of the series. [5]

5. A circle C_1 with centre A has equation

$$x^2 + y^2 - 6x + 8y - 75 = 0.$$

- (a) Find the coordinates of A and the radius of C_1 . [3]

- (b) A second circle C_2 has centre $B(-6, 8)$ and radius 5.

(i) Show that C_1 and C_2 touch.

(ii) Given that the circles touch at the point $P(-3, 4)$, find the equation of the common tangent. [7]

6. The triangle ABC is such that $AB = 6$ cm, $AC = 10$ cm and \widehat{BAC} is an **obtuse** angle. The area of triangle ABC is $15\sqrt{3}$ cm².

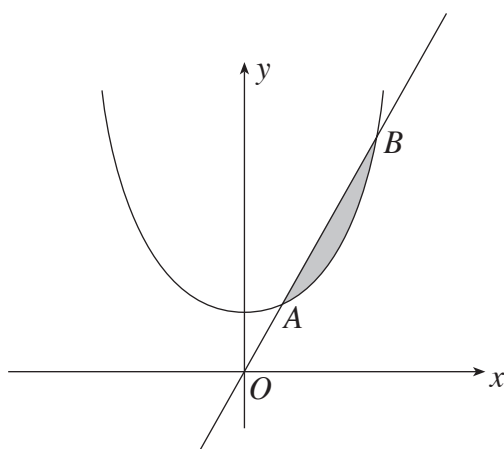
(a) Find the size of \widehat{BAC} . [3]

(b) Calculate the length of BC . [3]

7. (a) Find

$$\int \left(\sqrt{x} + \frac{2}{x^2} \right) dx. \quad [2]$$

(b)



The diagram shows a sketch of the curve $y = x^2 + 3$ and the line $y = 4x$. The line and the curve intersect at the points A and B .

(i) Showing your working, find the coordinates of A and B .

(ii) Evaluate the area of the shaded region. [10]

8. (a) Given that $x > 0$, $y > 0$, show that $\log_a(xy) = \log_a x + \log_a y$. [3]

(b) Express $\log_a 36 + \frac{1}{2} \log_a 256 - 2 \log_a 48$ as a single logarithm. [4]

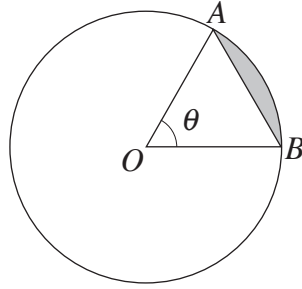
(c) Solve the equation

$$2^{x+1} = 5,$$

giving your answer correct to three decimal places. [2]

TURN OVER.

9.



The diagram shows two points A and B on a circle with centre O and radius 3 cm, such that $\widehat{AOB} = \theta$ radians. The perimeter of the **sector** AOB is 10 cm.

- (a) Find the value of θ . [3]
- (b) Find the area of the shaded segment, giving your answer correct to three decimal places. [4]