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MATHEMATICS M3

Mechanics 3

P.M. THURSDAY, 15 June 2006

($1\frac{1}{2}$ hours)

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

Take g as 9·8 ms $^{-2}$.

INFORMATION FOR CANDIDATES

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. A particle is moving along the x-axis. At time $t = 0$ the particle is at the origin O, its velocity is 12 ms^{-1} and its acceleration is -16 ms^{-2}. At time t s, the particle is x m from O and its velocity v ms^{-1} satisfies the equation

$$v = \frac{B}{x + A},$$

where A and B are constants.

(a) Find, in terms of A, B and x, an expression for the acceleration of the particle at time t s. [3]

(b) Show that $A = 9$ and $B = 108$. [3]

(c) Find an expression for t in terms of x. [4]

2. Find the general solution of the second order differential equation

$$\frac{\mathrm{d}^2x}{\mathrm{d}t^2} + 2\frac{\mathrm{d}x}{\mathrm{d}t} + 10x = 5t - 14,$$

such that $x = 4\frac{1}{2}$ and $\frac{\mathrm{d}x}{\mathrm{d}t} = 3\frac{1}{2}$ when $t = 0$. [12]

3. A ball of mass 0·4 kg is thrown vertically upwards from a point O with initial speed 17 ms^{-1}. When the ball is at a height of x m above O and its speed is v ms^{-1}, the air resistance acting on the ball has magnitude $0{\cdot}01\,v^2$ N.

(a) Show that, as the ball is ascending, v satisfies the differential equation

$$40v\frac{\mathrm{d}v}{\mathrm{d}x} = -(392 + v^2).$$ [3]

(b) Find, correct to two decimal places, the greatest height of the ball. [7]

(c) State, with a reason, whether the speed of the ball when it returns to O is

(i) greater than 17 ms^{-1},

(ii) less than 17 ms^{-1},

(iii) equal to 17 ms^{-1}. [2]

4. A particle P is moving in a straight line with Simple Harmonic Motion about centre O, with period 4 s. The maximum speed of P is 3π ms^{-1}. The point A is 4·8 m from O.

(a) Find the amplitude of the motion. [4]

(b) Find the speed of P when it is at A. [3]

(c) Calculate the time taken by P to move directly from O to A. [3]

(d) Determine the magnitude of the maximum acceleration of P. [2]

(e) Find the distance travelled by the particle P in 12 s. [2]

5. Particle A, of mass 2 kg, and particle B, of mass 3kg, are connected by a light inextensible string of length l m. Initially, both particles are lying at rest on a smooth horizontal surface a distance l m apart, with the string just slack. Particle B is given a blow of impulse 40 Ns in a direction away from A at an angle α to the line joining the initial positions of A and B.

Immediately after the blow, the speed of particle A is 4 ms^{-1}.

(a) Determine the value of α. [6]

(b) Calculate the magnitude and direction of the velocity of B immediately after the blow. [6]

6. A uniform ladder, of mass 37·5 kg and length 8 m, rests with its top end against a smooth vertical wall and its bottom end on rough horizontal ground. The ladder is inclined at an angle of 60° to ground. The coefficient of friction between the ladder and the ground is μ. A person, of mass 75 kg, climbs the ladder.

(a) Given that $\mu = 0{\cdot}25$, determine how far the person can ascend before the ladder slips. [10]

(b) Given that the person is able to ascend to the top of the ladder, determine the minimum value of μ. [4]

(c) State **one** modelling assumption you have made in your solution. [1]