WELSH JOINT EDUCATION COMMITTEE
General Certificate of Education
Advanced Subsidiary/Advanced

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496/01

MATHEMATICS P6

Pure Mathematics

P.M. FRIDAY, 27 January 2006

($1\frac{1}{2}$ hours)

LEGACY SPECIFICATION

ADDITIONAL MATERIALS

In addition to this examination paper, you will need:

- a 12 page answer book;
- a Formula Booklet;
- a calculator.

INSTRUCTIONS TO CANDIDATES

Answer **all** questions.

INFORMATION FOR CANDIDATES

Graphical calculators may be used for this paper.

The number of marks is given in brackets at the end of each question or part-question.

You are reminded of the necessity for good English and orderly presentation in your answers.

1. *(a)* Using the exponential definition of cosh x, show that

$$\cosh 2x = 2\cosh^2 x - 1.$$ [3]

(b) Solve the equation

$$\cosh 2x = 3\cosh x,$$

giving your answers correct to three significant figures. [8]

2. The equation

$$x^3 + x - 6 = 0$$

has a root α between 1 and 2.

(a) Show that the Newton-Raphson formula for finding the value of α can be written in the form

$$x_{n+1} = \frac{2x_n^{\,3} + 6}{3x_n^{\,2} + 1}\ .$$

[2]

(b) (i) Taking $x_0 = 1{\cdot}5$, find the values of x_1, x_2 and x_3 as accurately as your calculator will allow.

(ii) Round your value of x_3 to six decimal places and determine whether or not this gives the value of α correct to six decimal places. [5]

3. A parabola has parametric equations

$$x = t^2,\ y = 2t.$$

(a) Show that the length of the arc joining (0,0) to (1,2) is given by

$$2\int_0^1 \sqrt{t^2+1}\ \mathrm{d}t.$$

[3]

(b) Use the substitution $t = \sinh\theta$ to evaluate this arc length. [8]

4. Use the substitution $t = \tan\left(\dfrac{x}{2}\right)$ to evaluate the integral

$$\int_0^{\frac{\pi}{2}} \frac{\mathrm{d}x}{1+2\cos x}\ .$$ [8]

5. The function f is defined by

$$f(x) = \ln(1 + \sin x).$$

(a) Show that

$$f''(x) = -\frac{1}{1+\sin x}. \qquad [3]$$

(b) (i) Find the first four terms of the Maclaurin series of $f(x)$.

(ii) Deduce the first four terms of the Maclaurin series of the function g given by

$$g(x) = \ln(1 - \sin x).$$

(iii) By combining your series, show that

$$\ln\cos x = -\frac{x^2}{2} - \frac{x^4}{12} + \ldots \qquad [11]$$

6. Given that

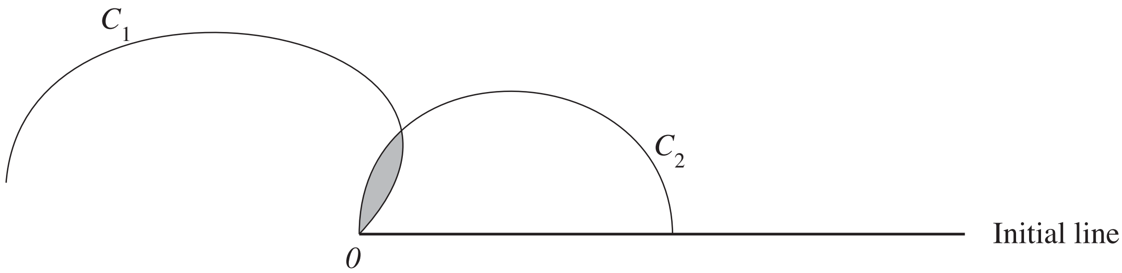
$$I_n = \int_0^4 x^n \sqrt{4-x}\, \mathrm{d}x \ (n \geqslant 0),$$

show that, for $n \geqslant 1$,

$$I_n = \frac{8n}{(2n+3)} I_{n-1}.$$

Hence evaluate I_2. [11]

7.



The diagram shows the curves C_1 and C_2 (not drawn to scale) with polar equations

C_1: $r = 1 - \cos\theta \qquad (0 \leqslant \theta \leqslant \pi)$,

C_2: $r = \cos\theta \qquad (0 \leqslant \theta \leqslant \frac{\pi}{2})$.

Find the area of the shaded region. [13]