

Transformations of graphs

Translations of graphs

A curve C_f has equation $y = f(x)$.

" a " is a positive number.

- The curve with equation $y = f(x) - b$ is the translation of C_f by vector $\begin{pmatrix} 0 \\ -b \end{pmatrix}$
- The curve with equation $y = f(x + a)$ is the translation of C_f by vector $\begin{pmatrix} -a \\ 0 \end{pmatrix}$



Combined translations

- The curve with equation $y + b = f(x + a)$ is the translation of C_f by vector $\begin{pmatrix} -a \\ -b \end{pmatrix}$

Examples: The curve with equation $y = (x - 3)^2 + 2$ is the translation of

the curve $y = x^2$ by vector $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$.

The circle $(x - 3)^2 + (y + 1)^2 = 9$ is the translation of the circle $x^2 + y^2 = 9$ by the vector $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$.

Parabolas

All parabolas of the form $y = x^2 + bx + c$ are the image of the parabola $y = x^2$

To work out the vector of this translation, use the completed square form:

$$y = x^2 + bx + c = (x + p)^2 + q$$

The vector of the translation is $\begin{pmatrix} -p \\ q \end{pmatrix}$.

Note: This vector is the vector \overrightarrow{OV} , where $V(-p, q)$ is the vertex of the parabola.

