

Simultaneous equations



SET OF LINEAR EQUATIONS

Consider the line $L_1 : ax + by = c$
and the line $L_2 : dx + ey = f$

To work out the coordinates of the point of INTERSECTION,
solve the equations SIMULTANEOUSLY.

Solving by *combination* / *elimination*:

$$\begin{cases} ax + by = c & (\times d) \\ dx + ey = f & (\times -a) \end{cases} \quad \begin{cases} adx + bdy = cd \\ -adx - aey = -af \end{cases}$$

Then add the equations to find the value of y . Use any other equation to find the value of x .

Solving by *identification*:

Make y the subject in both equations and identify the values of y :

$$L_1 : y = m_1x + c_1$$

$$L_2 : y = m_2x + c_2 \quad \text{this gives } (y =) m_1x + c_1 = m_2x + c_2 \text{ and solve.}$$

Solving by *substitution*:

Make y the subject in one of the equation then substitute y by this expression
in the second equation:

$$L_1 : y = mx + c$$

$$L_2 : dx + ey = f \quad \text{this gives } dx + e(mx + c) = f \text{ then solve.}$$

SET OF QUADRATIC AND LINEAR EQUATIONS



A parabola C has equation $y = ax^2 + bx + c$,

a line L has equation $y = dx + e$ (make y the subject if it is an implicit equation)

To work out the coordinates of the points of intersection
of the parabola and the line, solve these equations simultaneously

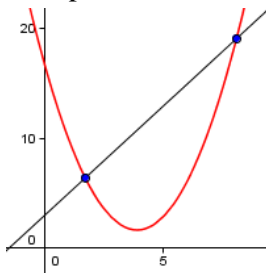
Solving by *identification*:

$$(y =) ax^2 + bx + c = dx + e \text{ then re-arrange into } ax^2 + (b-d)x + c - e = 0 \text{ and solve.}$$

Let's re-write as $Ax^2 + Bx + C = 0$

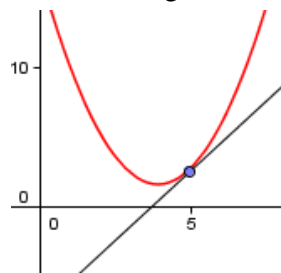
if $B^2 - 4AC > 0$

Two points of intersection



if $B^2 - 4AC = 0$

The line is tangent to the parabola



if $B^2 - 4AC < 0$

There is no intersection

