

Solution of linear equations – exam questions

Question 1: Jan 2006 – Q6

(a) Show that

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ bc & ca & ab \end{vmatrix} = (a-b)(b-c)(c-a) \quad (5 \text{ marks})$$

(b) (i) Hence, or otherwise, show that the system of equations

$$\begin{aligned} x + y + z &= p \\ 3x + 3y + 5z &= q \\ 15x + 15y + 9z &= r \end{aligned}$$

has no unique solution whatever the values of p , q and r . (2 marks)

(ii) Verify that this system is consistent when $24p - 3q - r = 0$. (2 marks)

(iii) Find the solution of the system in the case where $p = 1$, $q = 8$ and $r = 0$. (5 marks)

Question 2: Jun 2006 – Q5

A set of three planes is given by the system of equations

$$\begin{aligned} x + 3y - z &= 10 \\ 2x + ky + z &= -4 \\ 3x + 5y + (k-2)z &= k+4 \end{aligned}$$

where k is a constant.

(a) Show that $\begin{vmatrix} 1 & 3 & -1 \\ 2 & k & 1 \\ 3 & 5 & k-2 \end{vmatrix} = k^2 - 5k + 6$. (2 marks)

(b) In each of the following cases, determine the **number** of solutions of the given system of equations.

(i) $k = 1$.

(ii) $k = 2$.

(iii) $k = 3$. (7 marks)

(c) Give a geometrical interpretation of the significance of each of the three results in part (b) in relation to the three planes. (3 marks)

Question 3: Jan 2007 – Q1

Show that the system of equations

$$\begin{aligned} x + 2y - z &= 0 \\ 3x - y + 4z &= 7 \\ 8x + y + 7z &= 30 \end{aligned}$$

is inconsistent.

(4 marks)

Question 4: Jun 2007 – Q4

Consider the following system of equations, where k is a real constant:

$$\begin{aligned} kx + 2y + z &= 5 \\ x + (k+1)y - 2z &= 3 \\ 2x - ky + 3z &= -11 \end{aligned}$$

- (a) Show that the system does not have a unique solution when $k^2 = 16$. (3 marks)
- (b) In the case when $k = 4$, show that the system is inconsistent. (4 marks)
- (c) In the case when $k = -4$:
- (i) solve the system of equations; (5 marks)
- (ii) interpret this result geometrically. (1 mark)

Question 5: Jan 2008 – Q5

A system of equations is given by

$$\begin{aligned} x + 3y + 5z &= -2 \\ 3x - 4y + 2z &= 7 \\ ax + 11y + 13z &= b \end{aligned}$$

where a and b are constants.

- (a) Find the unique solution of the system in the case when $a = 3$ and $b = 2$. (5 marks)
- (b) (i) Determine the value of a for which the system does not have a unique solution. (3 marks)
- (ii) For this value of a , find the value of b such that the system of equations is consistent. (4 marks)

Question 6: Jun 2008 – Q6

Three planes have equations

$$\begin{aligned} x + y - 3z &= b \\ 2x + y + 4z &= 3 \\ 5x + 2y + az &= 4 \end{aligned}$$

where a and b are constants.

- (a) Find the coordinates of the single point of intersection of these three planes in the case when $a = 16$ and $b = 6$. (5 marks)
- (b) (i) Find the value of a for which the three planes do not meet at a single point. (3 marks)
- (ii) For this value of a , determine the value of b for which the three planes share a common line of intersection. (5 marks)

Question 7: Jan 2009 – Q7

Two fixed planes have equations

$$\begin{aligned}x - 2y + z &= -1 \\ -x + y + 3z &= 3\end{aligned}$$

- (a) The point P , whose z -coordinate is λ , lies on the line of intersection of these two planes. Find the x - and y -coordinates of P in terms of λ . (3 marks)
- (b) The point P also lies on the variable plane with equation $5x + ky + 17z = 1$. Show that

$$(k + 13)(2\lambda - 1) = 0 \quad (3 \text{ marks})$$

- (c) For the system of equations

$$\begin{aligned}x - 2y + z &= -1 \\ -x + y + 3z &= 3 \\ 5x + ky + 17z &= 1\end{aligned}$$

determine the solution(s), if any, of the system, and their geometrical significance in relation to the three planes, in the cases:

(i) $k = -13$;

(ii) $k \neq -13$. (6 marks)

Question 8: Jun 2009 - Q4

- (a) Show that the system of equations

$$\begin{aligned}3x - y + 3z &= 11 \\ 4x + y - 5z &= 17 \\ 5x - 4y + 14z &= 16\end{aligned}$$

does not have a unique solution and is consistent.

(You are not required to find any solutions to this system of equations.) (4 marks)

- (b) A transformation T of three-dimensional space maps points (x, y, z) onto image points (x', y', z') such that

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} x - y + 3z - 2 \\ 2x + 6y - 4z + 12 \\ 4x + 11y + 4z - 30 \end{bmatrix}$$

Find the coordinates of the invariant point of T . (8 marks)

Question 9: Jan 2010 – Q4

- (a) Determine the two values of k for which the system of equations

$$\begin{aligned}x - 2y + kz &= 5 \\(k + 1)x + 3y &= k \\2x + y + (k - 1)z &= 3\end{aligned}$$

does not have a unique solution. (4 marks)

- (b) Show that this system of equations is consistent for one of these values of k , but is inconsistent for the other.

(You are not required to find any solutions to this system of equations.) (8 marks)

Question 10: Jun 2010 – Q6

The line L and the plane Π have vector equations

$$\mathbf{r} = \begin{bmatrix} 7 \\ 8 \\ 50 \end{bmatrix} + t \begin{bmatrix} 6 \\ 2 \\ -9 \end{bmatrix} \quad \text{and} \quad \mathbf{r} = \begin{bmatrix} -2 \\ 0 \\ -25 \end{bmatrix} + \lambda \begin{bmatrix} 5 \\ 3 \\ 4 \end{bmatrix} + \mu \begin{bmatrix} 1 \\ 6 \\ 2 \end{bmatrix}$$

respectively.

- (a) (i) Find direction cosines for L . (2 marks)

- (ii) Show that L is perpendicular to Π . (3 marks)

- (b) For the system of equations

$$\begin{aligned}6p + 5q + r &= 9 \\2p + 3q + 6r &= 8 \\-9p + 4q + 2r &= 75\end{aligned}$$

form a pair of equations in p and q only, and hence find the unique solution of this system of equations. (5 marks)

- (c) It is given that L meets Π at the point P .

- (i) Demonstrate how the coordinates of P may be obtained from the system of equations in part (b). (2 marks)

- (ii) Hence determine the coordinates of P . (2 marks)

Solution of linear equations – exam questions MS

Question 1: Jan 2006 – Q6

<p>6(a) $\Delta = \begin{vmatrix} 0 & 0 & 1 \\ a-c & b-c & c \\ b(c-a) & a(c-b) & ab \end{vmatrix}$</p> <p>$= (a-c)(b-c) \begin{vmatrix} 0 & 0 & 1 \\ 1 & 1 & c \\ -b & -a & ab \end{vmatrix}$</p> <p>$= (a-c)(b-c)(b-a)$</p> <p>$= (a-b)(b-c)(c-a)$</p>	M1				
<p>b(i) Noting $a = b = 3$, $c = 5$ in coefft. matrix $\Rightarrow \Delta = 0$ since factor $(a-b) = 0$</p> <p>Hence no unique soln. to system</p>	A1 A1	5			
<p>(ii) $3R_2 + R_3 = 24R_1$</p> <p>giving consistency iff $24p - 3q - r = 0$</p>	M1 A1	2			
<p>(iii) $x + y + z = 1$ $3x + 3y + 5z = 8$ $5x + 5y + 3z = 0$</p> <p>$(2) - 3 \times (1)$ or $(3) - 5 \times (1) \Rightarrow z = 2\frac{1}{2}$</p> <p>Then $x + y = -1\frac{1}{2}$</p> <p>Setting $x = \lambda$ (e.g.) $\Rightarrow y = -1\frac{1}{2} - \lambda$</p> <p>All correct, any form</p> <p><u>Special Case</u> ruling for those who don't attempt to parametrise but who show this is the full solution: 4 + B0</p>	M1 A1 B1 M1 A1	5			
Total					12

Question 2: Jun 2006 – Q5

<p>5(a) $\begin{vmatrix} 1 & 3 & -1 \\ 2 & k & 1 \\ 3 & 5 & k-2 \end{vmatrix}$</p> <p>$= k^2 - 2k - 10 + 9 + 3k - 5 - 6(k-2)$</p> <p>$= k^2 - 5k + 6$</p>	M1				
<p>b(i) When $k = 1$, $\Delta \neq 0 \Rightarrow \exists$ one soln.</p>	A1 B1	2			
<p>(ii) When $k = 2$, $\Delta = 0$ (no unique soln.)</p> <p>System is $x + 3y - z = 10$ $2x + 2y + z = -4$ $3x + 5y = 6$</p> <p>$R_1 + R_2 = R_3$ on both sides</p> <p>$\Rightarrow \exists \infty$-ly many soln.s</p>	B1 M1 A1	7			
<p>(iii) When $k = 3$, $\Delta = 0$ (no unique soln.)</p> <p>System is $x + 3y - z = 10$ $2x + 3y + z = -4$ $3x + 5y + z = 7$</p> <p>$(1) + (2) \Rightarrow 3x + 6y = 6$ $(1) + (3) \Rightarrow 4x + 8y = 17$</p> <p>$\Rightarrow$ System inconsistent and \exists no soln.s</p>	B1 M1 A1	3			
<p>(c) $k = 1$: the (single) point of intersection of 3 planes</p> <p>$k = 2$: 3 planes meet in a line (or form a sheaf)</p> <p>$k = 3$: 3 planes form a "prism" (or have three parallel lines of intersection; or have no common intersection)</p>	B1✓ B1✓ B1✓	3			
Total					12

Question 3: Jan 2007 – Q1

<p>1 $\begin{vmatrix} 1 & 2 & -1 & 0 & 1 & 2 & -1 & 0 \\ 3 & -1 & 4 & 7 & \rightarrow & 0 & -1 & 7 & 7 \\ 8 & 1 & 7 & 30 & 0 & -15 & 15 & 30 \end{vmatrix}$</p> <p>$\begin{vmatrix} 1 & 2 & -1 & 0 \\ \rightarrow & 0 & -1 & 1 & 1 \\ 0 & -1 & 1 & 2 \end{vmatrix}$</p>	M1 A1				
<p>Or $\Delta = -7 - 3 + 64 - 8 - 4 - 42 = 0$</p> <p>and Δ_x or Δ_y or $\Delta_z = 0$ shown also</p> <p>Explaining this \Rightarrow inconsistency</p>	(M1) (A1) (B1)	4			
<p>Or Solving (1) & (2), say, to get $x = \lambda$, $y = 1 - \lambda$, $z = 2 - \lambda$</p> <p>Substⁿ. in (3) $\Rightarrow 15 = 30$</p>	(M1) (A1) (A1) (B1)	(4)			
Total					4

Question 4: Jun 2007 – Q4

<p>4(a) $\Delta = \begin{vmatrix} k & 2 & 1 \\ 1 & k+1 & -2 \\ 2 & -k & 3 \end{vmatrix}$</p> <p>$= 3k^2 + 3k - k - 8 - 2(k+1) - 2k^2 - 6$</p> <p>$= k^2 - 16$</p> <p>When $k^2 = 16$ $\Delta = 0 \Rightarrow$ no unique soln.</p> <p>Or Substⁿ. Both $k = 4$ and $k = -4$ and attempt at det.</p> <p>Each case correctly shown</p>	M1 A1 E1				
<p>(b) $4x + 2y + z = 5$ $k = 4 \Rightarrow x + 5y - 2z = 3$ $2x - 4y + 3z = -11$</p> <p>Elimⁿ. z from (1) & (2) $\Rightarrow 9(x+y) = 13$ $(1) \& (3) \Rightarrow 10(x+y) = 26$ Or $(2) \& (3) \Rightarrow 7(x+y) = -13$</p> <p>Explaining inconsistency, eg from $\frac{13}{9} \neq \frac{26}{10}$</p>	B1 M1 A1 E1	3			
<p>Alternatively (mark as above)</p> <p>Elimⁿ. x from (1) & (2) $\Rightarrow 9(2y-z) = 7$ $(2) \& (3) \Rightarrow 7(2y-z) = 17$ $(1) \& (3) \Rightarrow 5(2y-z) = 27$</p> <p>Or</p> <p>Elimⁿ. y from (1) & (2) $\Rightarrow 9(2x+z) = 19$ $(2) \& (3) \Rightarrow 7(2x+z) = -43$ $(1) \& (3) \Rightarrow 5(2x+z) = -1$</p>	(M1) (A1) (A1)	3			
<p>(c)(i) $-4x + 2y + z = 5$ $k = -4 \Rightarrow x - 3y - 2z = 3$ $2x + 4y + 3z = -11$</p> <p>Eliminating one variable $-7x + y = 13$ Or $10y + 7z = -17$ Or $10x + z = -21$</p> <p>Parametrisation</p> <p>$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 13 \\ -21 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 7 \\ -10 \end{pmatrix}$</p>	B1 M1 A1 M1	5			
<p>Correct alternate answer forms: $x, y = 13 + 7\lambda, z = -21 - 10\lambda$ $y, x = (y-13)/7, z = (-21-10y)/7$ $z, y = (-17-7z)/10, x = (-21-z)/10$</p> <p>Do not accept a mixed parametrisation</p>					
<p>(ii) The line of intersection of 3 planes</p>	B1	1			
Total					13

Question 5: Jan 2008 – Q5

<p>5(a) eg $3 \times (1) - (2) \Rightarrow 13y + 13z = -13$ $(3) - (2) \Rightarrow 15y + 11z = -5$</p> <p>$x = 6, y = 1\frac{1}{2}, z = -2\frac{1}{2}$</p> <p>Alt I (Cramer's Rule):</p> $\Delta = \begin{vmatrix} 1 & 3 & 5 \\ 3 & -4 & 2 \\ 3 & 11 & 13 \end{vmatrix}, \Delta_x = \begin{vmatrix} -2 & 3 & 5 \\ 7 & -4 & 2 \\ 2 & 11 & 13 \end{vmatrix},$ $\Delta_y = \begin{vmatrix} 1 & -2 & 5 \\ 3 & 7 & 2 \\ 3 & 2 & 13 \end{vmatrix}, \Delta_z = \begin{vmatrix} 1 & 3 & -2 \\ 3 & -4 & 7 \\ 3 & 11 & 2 \end{vmatrix}$ <p>$= 52, 312, 78$ and -130 respectively</p> $x = \frac{\Delta_x}{\Delta}, y = \frac{\Delta_y}{\Delta}, z = \frac{\Delta_z}{\Delta}$ <p>$x = 6, y = 1\frac{1}{2}, z = -2\frac{1}{2}$</p> <p>Alt II (Augmented matrix method):</p> $\left[\begin{array}{ccc c} 1 & 3 & 5 & -2 \\ 3 & -4 & 2 & 7 \\ 3 & 11 & 13 & 2 \end{array} \right] \rightarrow$ $\left[\begin{array}{ccc c} 1 & 3 & 5 & -2 \\ 0 & -13 & -13 & 13 \\ 0 & 2 & -2 & 8 \end{array} \right]$ $\rightarrow \left[\begin{array}{ccc c} 1 & 3 & 5 & -2 \\ 0 & 1 & 1 & -1 \\ 0 & 1 & -1 & 4 \end{array} \right]$ $\rightarrow \left[\begin{array}{ccc c} 1 & 3 & 5 & -2 \\ 0 & 1 & 1 & -1 \\ 0 & 0 & -2 & 5 \end{array} \right]$ <p>Substituting back to get $x = 6, y = 1\frac{1}{2}, z = -2\frac{1}{2}$</p> <p>Alt III (Inverse matrix method):</p> $C^{-1} = \frac{1}{52} \begin{bmatrix} -74 & 16 & 26 \\ -33 & -2 & 13 \\ 45 & -2 & -13 \end{bmatrix}$ $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = C^{-1} \begin{bmatrix} -2 \\ 7 \\ 2 \end{bmatrix} = \begin{bmatrix} 6 \\ 1.5 \\ -2.5 \end{bmatrix}$	<p>M1 A1A1 M1 A1</p> <p>(M1)</p> <p>(A1 A1)</p> <p>(M1)</p> <p>(A1)</p> <p>(M1 A1)</p> <p>(M1) (A1)</p> <p>(M1) (A1)</p>	<p>5</p> <p>(5)</p> <p>(5)</p> <p>(5)</p>
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Question 6: Jun 2008 – Q6

<p>6(a) eg $(2) - (1) \Rightarrow x + 7z = -3$ $(3) - 2 \times (2) \Rightarrow x + 8z = -2$ Solving 2×2 system $x = -10, y = 19, z = 1$</p> <p>(b)(i) $\begin{vmatrix} 1 & 1 & -3 \\ 2 & 1 & 4 \\ 5 & 2 & a \end{vmatrix} = 15 - a$</p> <p>Setting = to zero and solving for a $a = 15$</p> <p>$x + y - 3z = b$ $2x + y + 4z = 3$ $5x + 2y + 15z = 4$</p> <p>(ii) eg $(2) - (1) \Rightarrow x + 7z = 3 - b$ $(3) - 2 \times (2) \Rightarrow x + 7z = -2$ Equating the two RHSs $b = 5$</p>	<p>M1A1 A1 M1 A1</p> <p>B1</p> <p>M1 A1</p> <p>M1A1 A1 M1 A1</p>	<p>5</p> <p>3</p> <p>5</p>
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Total	13	
Alternate Schemes		
<p>6(a) Cramer's Rule</p> $\Delta = \begin{vmatrix} 1 & 1 & -3 \\ 2 & 1 & 4 \\ 5 & 2 & 16 \end{vmatrix}, \Delta_x = \begin{vmatrix} 6 & 1 & -3 \\ 3 & 1 & 4 \\ 4 & 2 & 16 \end{vmatrix}$ $\Delta_y = \begin{vmatrix} 1 & 6 & -3 \\ 2 & 3 & 4 \\ 5 & 4 & 16 \end{vmatrix}, \Delta_z = \begin{vmatrix} 1 & 1 & 6 \\ 2 & 1 & 3 \\ 5 & 2 & 4 \end{vmatrix}$ <p>$= -1, 10, -19$ and -1 respectively</p> $x = \frac{\Delta_x}{\Delta}, y = \frac{\Delta_y}{\Delta}, z = \frac{\Delta_z}{\Delta}$ <p>$x = -10, y = 19, z = 1$</p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1 A1</p>	<p>(5)</p>

Question 7: Jan 2009 – Q7

<p>7(a) $x - 2y = -1 - \lambda$ $-x + y = 3 - 3\lambda$ Solving for x and y in terms of λ $x = 7\lambda - 5$ and $y = 4\lambda - 2$</p> <p>(b) Substⁿ. x, y, z in terms of λ in $5x + ky + 17z = 1$ $35\lambda - 25 + k(4\lambda - 2) + 17\lambda - 1 = 0$ Factsn. attempt: $(4y - 2)(k + 13) = 0$ $(2y - 1)(k + 13) = 0$</p> <p>c(i) When $k = -13, 5x - 13y + 17z = 1$ $= 35\lambda - 25 - 52\lambda + 26 + 17\lambda = 1$</p> <p>The three planes intersect in a line Solns. $x = 7\lambda - 5, y = 4\lambda - 2, z = \lambda$</p> <p>(ii) When $k \neq -13, \lambda = \frac{1}{2}$ Soln. $(-\frac{1}{2}, 0, \frac{1}{2})$ Three planes meet at a point</p>	<p>B1 M1 A1</p> <p>M1</p> <p>dM1 A1</p> <p>B1</p> <p>B1 B1F</p> <p>B1 B1F B1</p>	<p>3</p> <p>3</p> <p>6</p>
Total		12

Question 8: Jun 2009 - Q4

4(a)	$3 \times [1] - [2] \Rightarrow 5x - 4y + 14z = 16$	M2 A1	
	Giving no unique soln. <i>and</i> consistent	E1	
	For those who just show $\Delta = 0$ to conclude that there is no unique soln.	(M1)	
	OR	(A1)	
	Solving e.g. in [1] & [2]:	(M1)	
	$\frac{x-4}{2} = \frac{y-1}{27} = \frac{z}{7} = \lambda$	(A1)	
	Subst ⁿ . in [3] for x, y, z in terms of λ	(M1)	
	Showing LHS = RHS = 16	(A1)	
	OR		
	$3 \begin{array}{ccc ccc} -1 & 3 & 11 & 3 & -1 & 3 \\ 1 & -5 & 17 & 1 & 2 & -8 \\ -4 & 14 & 16 & -1 & -2 & 8 \end{array} \begin{array}{c} 1 \\ 6 \\ -6 \end{array}$	(M1)	
		(A1)	
		(A1)	
	$R_2' = -R_3' \Rightarrow$ no unique soln. and consistency	(E1)	
	OR		
	Showing $\Delta = 0 \Rightarrow$ no unique soln.	(M1)	
		(A1)	
	Attempt at each of $\Delta_x = \begin{vmatrix} 11 & -1 & 3 \\ 17 & 1 & -5 \\ 16 & -4 & 14 \end{vmatrix}$,		
	$\Delta_y = \begin{vmatrix} 3 & 11 & 3 \\ 4 & 17 & -5 \\ 5 & 16 & 14 \end{vmatrix}$ and $\Delta_z = \begin{vmatrix} 3 & -1 & 11 \\ 4 & 1 & 17 \\ 5 & -4 & 16 \end{vmatrix}$	(M1)	
	Each shown = 0 and this \Rightarrow consistency	(A1)	4
(b)	Setting $x' = x, y' = y, z' = z$	M1	
	$2 = -y + 3z$		
	$-12 = 2x + 5y - 4z$	A1	
	$30 = 4x + 11y + 3z$		
	E.g. $\left. \begin{array}{l} 2 = 3z - y \\ 54 = 11z + y \end{array} \right\}$ by $(3) - 2 \times (2)$	M1	
	$z = 4, y = 10$	A1	
	$x = -23$	M1 A1	
	OR	M1 A1	8
	Other methods for solving a 3×3 system will be constructed should they arise		
	Total		12

Question 9: Jan 2010 - Q4

4(a)	$\begin{vmatrix} 1 & -2 & k \\ k+1 & 3 & 0 \\ 2 & 1 & k-1 \end{vmatrix} = 3k^2 - 2k - 5$	M1	
		A1	
	$k = \frac{5}{3}, -1$	M1	
		A1	4
(b)	$5 = x - 2y + \frac{5}{3}z$		
	$k = \frac{5}{3} \Rightarrow \frac{5}{3} = \frac{8}{3}x + 3y$	B1	
	$3 = 2x + y + \frac{2}{3}z$		
	$8x + 9y = 5 / 15y - 8z = -21 / 5x + 3z = 11$	A1; A1	
	$5 = x - 2y - z$	M1	
	$k = -1 \Rightarrow -1 = 3y$	B1	
	$3 = 2x + y - 2z$	M1	
	$x - z = \frac{13}{3} / 2x - 2z = \frac{10}{3} / 5x - 5z = 11$		
	OR	A1	
	$y = -\frac{1}{3}$ and $y = -\frac{7}{5}$ found	A1	8
	Total		12

Question 10: Jun 2010 - Q6

6(a)(i)	$\bullet = \sqrt{6^2 + 2^2 + 9^2}$ attempted <i>and</i>		
	$\pm \left(\frac{6}{\bullet}, \frac{2}{\bullet}, \frac{-9}{\bullet} \right)$	M1	
	$\bullet = 11$ and all correct	A1	2
(ii)	Either $\begin{bmatrix} 5 \\ 3 \\ 4 \end{bmatrix} \times \begin{bmatrix} 1 \\ 6 \\ 2 \end{bmatrix} = \begin{bmatrix} 6 \\ -3 \\ -9 \end{bmatrix}$	M1	
	Explaining that d.v. of L is in dirn. of II 's nml. $\Rightarrow L \perp II$	A1	
		B1	
	Or $\begin{bmatrix} 6 \\ 2 \\ -9 \end{bmatrix} \bullet \begin{bmatrix} 5 \\ 3 \\ 4 \end{bmatrix} = 0$ and $\begin{bmatrix} 6 \\ 2 \\ -9 \end{bmatrix} \bullet \begin{bmatrix} 1 \\ 6 \\ 2 \end{bmatrix} = 0$	(M1)	
		(A1)	
	Explaining that d.v. of L is \perp to 2 (non-//) vectors in $II \Rightarrow L \perp II$	(B1)	3
(b)	E.g. $6 \times \textcircled{1} - \textcircled{2}: 46 = 34p + 27q$	M1	
	$2 \times \textcircled{1} - \textcircled{3}: -57 = 21p + 6q$	A1 A1	
	$\textcircled{2} - 3 \times \textcircled{3}: -217 = 29p - 9q$		
	$2 \times \textcircled{4} + 9 \times \textcircled{5}: 605 = -121p$	M1	
	$p = -5, q = 8, r = -1$	A1	5
(c)	$7 + 6t = -2 + 5\lambda + \mu$		
(i)	$8 + 2t = 0 + 3\lambda + 6\mu$		
	$50 - 9t = -25 + 4\lambda + 2\mu$	M1	
	$9 = -6t + 5\lambda + \mu$		
	$\rightarrow 8 = -2t + 3\lambda + 6\mu$		
	$75 = 9t + 4\lambda + 2\mu$		
	i.e. the above system with $p = -t, q = \lambda$ and $r = \mu$	A1	2
(ii)	Subst ⁿ . $t = 5$ into L 's eqn.		
	Or $\lambda = 8$ and $\mu = -1$ into II 's eqn.	M1	
	$P = (37, 18, 5)$	A1	2
	Total		14