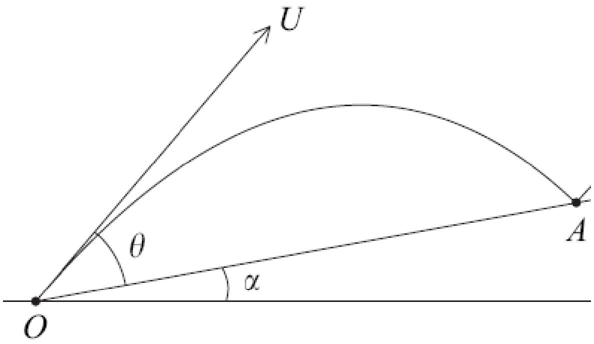
Projectiles on inclined planed – exam questions

Question 1: June 2006 – Q7

A projectile is fired from a point O on the slope of a hill which is inclined at an angle α to the horizontal. The projectile is fired up the hill with velocity U at an angle θ above the hill and first strikes it at a point A. The projectile is modelled as a particle and the hill is modelled as a plane with OA as a line of greatest slope.

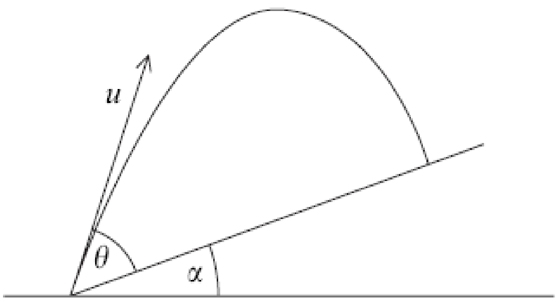
(a) (i) Find, in terms of U, g, α and θ, the time taken by the projectile to travel from O to A. *(3 marks)*

(ii) Hence, or otherwise, show that the magnitude of the component of the velocity of the projectile perpendicular to the hill, when it strikes the hill at the point A, is the same as it was initially at O. *(3 marks)*



Question 2: June 2007 – Q7

A particle is projected from a point on a plane which is inclined at an angle α to the horizontal. The particle is projected up the plane with velocity u at an angle θ above the plane. The motion of the particle is in a vertical plane containing a line of greatest slope of the inclined plane.



(a) Using the identity $\cos(A+B) = \cos A \cos B - \sin A \sin B$, show that the range up the plane is

$$\frac{2u^2 \sin\theta \cos(\theta+\alpha)}{g\cos^2\alpha}$$ *(8 marks)*

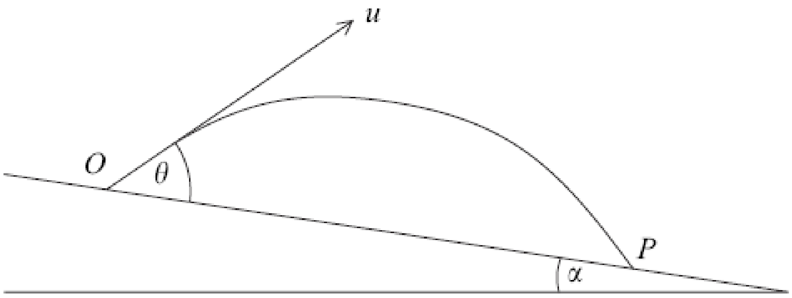
(b) Hence, using the identity $2\sin A \cos B = \sin(A+B) + \sin(A-B)$, show that, as θ varies, the range up the plane is a maximum when $\theta = \frac{\pi}{4} - \frac{\alpha}{2}$. *(3 marks)*

(c) Given that the particle strikes the plane at right angles, show that

$$2\tan\theta = \cot\alpha$$ *(4 marks)*

Question 3: June 2008 – Q7

A projectile is fired with speed u from a point O on a plane which is inclined at an angle α to the horizontal. The projectile is fired at an angle θ to the inclined plane and moves in a vertical plane through a line of greatest slope of the inclined plane. The projectile lands at a point P, lower down the inclined plane, as shown in the diagram.



(a) Find, in terms of u, g, θ and α, the greatest perpendicular distance of the projectile from the plane. *(4 marks)*

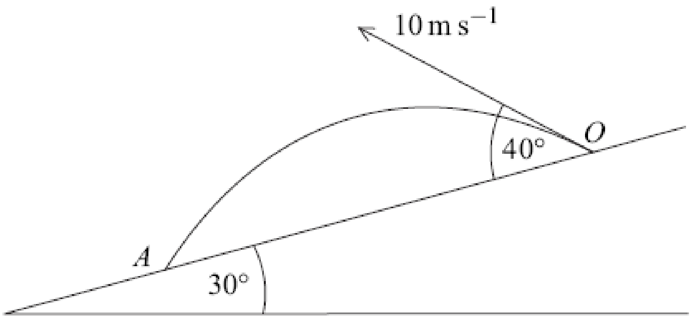
(b) (i) Find, in terms of u, g, θ and α, the time of flight from O to P. *(2 marks)*

(ii) By using the identity $\cos A \cos B + \sin A \sin B = \cos(A - B)$, show that the distance OP is given by $\dfrac{2u^2 \sin\theta \cos(\theta - \alpha)}{g\cos^2\alpha}$. *(6 marks)*

(iii) Hence, by using the identity $2\sin A \cos B = \sin(A + B) + \sin(A - B)$ or otherwise, show that, as θ varies, the maximum possible distance OP is $\dfrac{u^2}{g(1 - \sin\alpha)}$. *(5 marks)*

Question 4: June 2009 – Q7

A particle is projected from a point O on a smooth plane which is inclined at $30°$ to the horizontal. The particle is projected down the plane with velocity $10\,\text{m s}^{-1}$ at an angle of $40°$ above the plane and first strikes it at a point A. The motion of the particle is in a vertical plane containing a line of greatest slope of the inclined plane.



(a) Show that the time taken by the particle to travel from O to A is

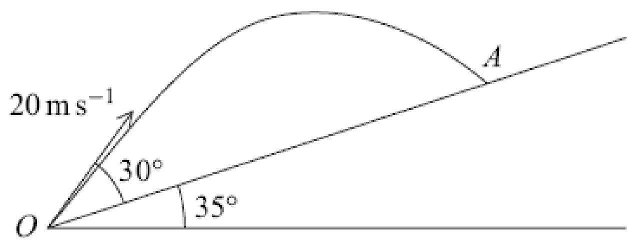
$$\frac{20\sin 40°}{g\cos 30°}$$ *(3 marks)*

(b) Find the components of the velocity of the particle parallel to and perpendicular to the slope as it hits the slope at A. *(4 marks)*

(c) The coefficient of restitution between the slope and the particle is 0.5. Find the speed of the particle as it rebounds from the slope. *(4 marks)*

Question 5: June2010 – Q7

A ball is projected from a point O on a smooth plane which is inclined at an angle of $35°$ above the horizontal. The ball is projected with velocity 20 m s^{-1} at an angle of $30°$ above the plane, as shown in the diagram. The motion of the ball is in a vertical plane containing a line of greatest slope of the inclined plane. The ball strikes the inclined plane at the point A.



(a) Find the components of the velocity of the ball, parallel and perpendicular to the plane, as it strikes the inclined plane at A. *(7 marks)*

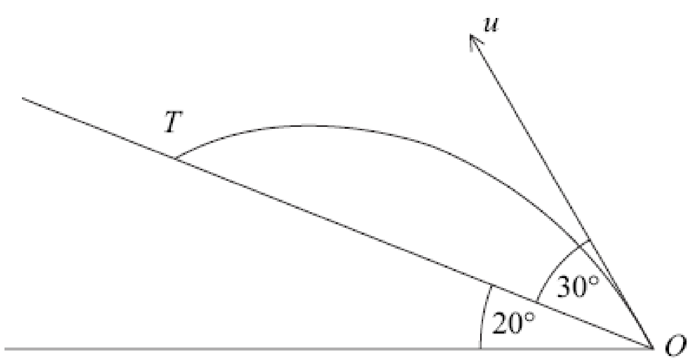
(b) On striking the plane at A, the ball rebounds. The coefficient of restitution between the plane and the ball is $\frac{4}{5}$.

Show that the ball next strikes the plane at a point lower down than A. *(6 marks)*

Question 6: June 2011 – Q6

A projectile is fired from a point O on a plane which is inclined at an angle of $20°$ to the horizontal. The projectile is fired up the plane with velocity $u\text{ m s}^{-1}$ at an angle of $30°$ to the inclined plane. The projectile travels in a vertical plane containing a line of greatest slope of the inclined plane.

The projectile hits a target T on the inclined plane.

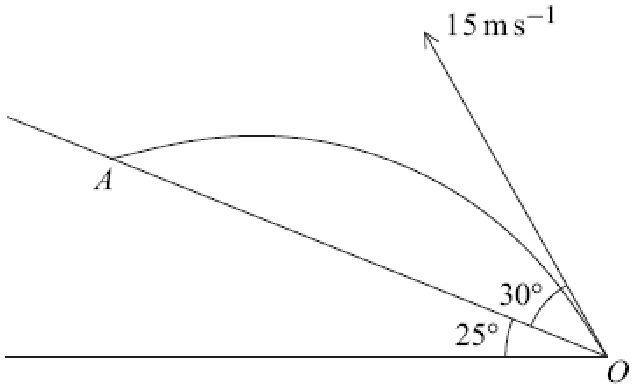


(a) Given that $OT = 200\text{ m}$, determine the value of u. *(7 marks)*

(b) Find the greatest perpendicular distance of the projectile from the inclined plane. *(4 marks)*

Question 7: June 2012 – Q5

A particle is projected from a point O on a smooth plane, which is inclined at $25°$ to the horizontal. The particle is projected up the plane with velocity $15\,\text{m s}^{-1}$ at an angle $30°$ above the plane. The particle strikes the plane for the first time at a point A. The motion of the particle is in a vertical plane containing a line of greatest slope of the inclined plane.



(a) Find the time taken by the particle to travel from O to A. *(4 marks)*

(b) The coefficient of restitution between the particle and the inclined plane is $\frac{2}{3}$.

Find the speed of the particle as it rebounds from the inclined plane at A. *(8 marks)*

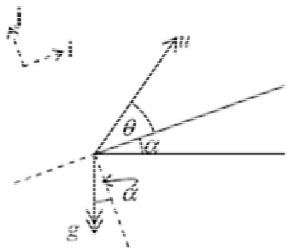
Projectiles on inclined planed – exam questions MS

Question 1: June 2006 – Q7

(a)(i)	the projectile hits the plane again when		
	$(Ut\sin\theta - \frac{1}{2}gt^2\cos\alpha) = 0$	M1A1	
	$\therefore t = \frac{2U\sin\theta}{g\cos\alpha}$	A1F	3
(ii)	the component of velocity perpendicular to plane =		
	$U\sin\theta - g\frac{2U\sin\theta}{g\cos\alpha}\cos\alpha =$	M1A1F	
	$-U\sin\theta =$		
	the initial magnitude	A1	3

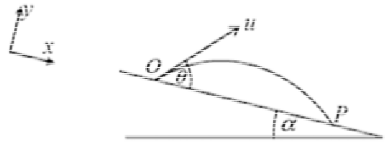
Question 2: June 2007 – Q7

(a)



(a)	$y = ut\sin\theta - \frac{1}{2}gt^2\cos\theta$	M1A1	
	$y = 0 \quad \Rightarrow \quad t = \frac{2u\sin\theta}{g\cos\alpha}$	A1F	
	$x = ut\cos\theta - \frac{1}{2}gt^2\sin\alpha$	M1A1	
	$R = u\frac{2u\sin\theta}{g\cos\alpha}\cos\theta - \frac{1}{2}g\left(\frac{2u\sin\theta}{g\cos\alpha}\right)^2\sin\alpha$	M1	
	$R = \frac{2u^2\sin\theta\cos(\theta+\alpha)}{g\cos^2\alpha}$	m1 A1	8
(b)	$R = \frac{2u^2\times\frac{1}{2}[\sin(2\theta+\alpha)+\sin(-\alpha)]}{g\cos^2\alpha}$	B1	
	R is maximum when $\sin(2\theta+\alpha) = 1$	M1	
	or $2\theta+\alpha = \frac{\pi}{2}$		
	$\therefore \quad \theta = \frac{\pi}{4} - \frac{\alpha}{2}$	A1	3
(c)	$y = 0 \quad \Rightarrow \quad t = \frac{2u\sin\theta}{g\cos\alpha}$	M1 A2,1	
	$\dot{x} = 0 \quad \Rightarrow \quad t = \frac{u\cos\theta}{g\sin\alpha}$		
	$\frac{2u\sin\theta}{g\cos\alpha} = \frac{u\cos\theta}{g\sin\alpha}$		
	$2\tan\theta = \cot\alpha$	A1	4
	Total		15

Question 3: June 2008 – Q7



(a)	$v_y^2 = u^2\sin^2\theta - 2g\cos\alpha . y$	M1 A1	
	$0 = u^2\sin^2\theta - 2g\cos\alpha . y_{max}$	m1	
	$y_{max} = \frac{u^2\sin^2\theta}{2g\cos\alpha}$	A1F	4
(b)(i)	$u\sin\theta\, t - \frac{1}{2}g\cos(\alpha)t^2 = 0$	M1	
	$t = \frac{2u\sin\theta}{g\cos\alpha}$	A1	2
(ii)	$x = u\cos\theta t - \frac{1}{2}g\sin(-\alpha)t^2$	M1 A1	
	$R = u\cos\theta(\frac{2u\sin\theta}{g\cos\alpha}) + \frac{1}{2}g\sin\alpha(\frac{2u\sin\theta}{g\cos\alpha})^2$	M1	
	$= \frac{2u^2\cos\theta\sin\theta\cos\alpha + 2u^2\sin\alpha\sin^2\theta}{g\cos^2\alpha}$	m1	
	$= \frac{2u^2\sin\theta(\cos\theta\cos\alpha + \sin\theta\sin\alpha)}{g\cos^2\alpha}$	A1F	
	$= \frac{2u^2\sin\theta\cos(\theta-\alpha)}{g\cos^2\alpha}$	A1	6
(iii)	$\overline{OP} = \frac{2u^2\sin\theta\cos(\theta-\alpha)}{g\cos^2\alpha}$		
	$= \frac{2u^2\frac{1}{2}[\sin(2\theta-\alpha)+\sin\alpha]}{g\cos^2\alpha}$	M1A1	
	$\overline{OP}$ is max when $\sin(2\theta-\alpha) = 1$	M1	
	$\overline{OP}_{max} = \frac{u^2(1+\sin\alpha)}{g\cos^2\alpha}$	A1F	
	$\overline{OP}_{max} = \frac{u^2(1+\sin\alpha)}{g(1-\sin^2\alpha)}$		
	$\overline{OP}_{max} = \frac{u^2}{g(1-\sin\alpha)}$	A1	5
	Total		17

Question 4: June 2009 – Q7

a)	$y = 10t\sin 40° - \frac{1}{2}gt^2\cos 30°$	M1A1	
	$y = 0 \Rightarrow t = \frac{20\sin 40°}{g\cos 30°}$	A1	3
b)	$\dot{x} = 10\cos 40° + g\sin 30°\left(\frac{20\sin 40°}{g\cos 30°}\right)$	M1	
	$\dot{x} = 15.08 \text{ ms}^{-1}$	A1	
	$\dot{y} = 10\sin 40° - g\cos 30°\left(\frac{20\sin 40°}{g\cos 30°}\right)$	M1	
	$\dot{y} = -6.427 \text{ ms}^{-1}$	A1	4
c)	$\dot{x}$ will be unchanged	B1	
	Rebound $\dot{y} = 6.427\times 0.5 = 3.214$	M1	
	Rebound speed $= \sqrt{15.08^2 + 3.214^2}$	m1	
	$= 15.4 \text{ ms}^{-1}$	A1F	4
	Total		11

Question 5: June2010 – Q7

(a)	On striking A:		
	$20\sin30^\circ.t-\frac{1}{2}(9.8)\cos35^\circ.t^2=0$	M1A1	
	$t=2.49$	A1	
	Components of Velocity :		
	$u_x=20\cos30^\circ-9.8\sin35^\circ(2.49)$	M1	
	$u_x=3.32$	A1F	
	$u_y=20\sin30^\circ-9.8\cos35^\circ(2.49)$	M1	
	$u_y=-10$ (or -9.99)	A1F	7
(b)	On Rebounding		
	$v_x=3.32$		
	$v_y=\frac{4}{5}\times10$	B1F	
	$v_y=8$ (or 7.99)		
	The rebound angle $=\tan^{-1}\frac{8}{3.32}$	M1	
	$=67.5^\circ$ (or 67.4°)	A1F	
	$35^\circ+67.5^\circ=102.5^\circ$	M1 A1F	
	$102.5^\circ>90^\circ$, therefore the second strike will be at a point lower down than A.	E1	6
	Alternative:		
	$\frac{4}{5}\times10=8$	(B1)	
	$0=8t-\frac{1}{2}g\cos35\,t^2$	(M1)	
	$t=1.9931$	(A1)	
	$x=3.32t-\frac{1}{2}g\sin35\,t^2$	(M1)	
	$x=-4.55$ or -4.56	(A1)	
	The second strike will be at a point lower down than A.	(E1)	
	Total		**13**

Question 6: June 2011 – Q6

6 (a)	Perpendicular to the plane:		
	$y=-\frac{1}{2}gt^2\cos20+ut\sin30$	M1	
	$0=-4.9t^2\cos20+ut\sin30$	M1	
	$t=0.108589568u$ or $\frac{2u\sin30}{g\cos20}$	A1	
	Parallel to the plane:		
	$x=-\frac{1}{2}gt^2\sin20+ut\cos30$	M1	
	$200=-4.9(0.108589568u)^2\sin20+u(0.108589568u)\cos30$	m1	
	$u^2=2693$	A1F	
	$u=51.9$ or 51.894	A1F	7
(b)	$\dot{y}=-gt\cos20+u\sin30=0$	M1	
	$t=2.817899$ or 2.817580214 or $\frac{51.9\sin30}{g\cos20}$	A1F	
	The greatest $\perp$ distance =		
	$-\frac{1}{2}9.8(2.817899)^2\cos20+51.9(2.817899)\sin30$ or $-\frac{1}{2}9.8(\frac{51.894\sin30}{9.8\cos20})^2\cos20+51.9(\frac{51.894\sin30}{9.8\cos20})\sin30$	m1	
	$=36.5622$ m or 36.5538 $=36.6$ 3sf	A1F	4
			11
6 (a)	**Alternative:**		
	$x=200\cos20$	B1	
	$y=200\sin30$	B1	
	$200\cos20=u\cos50t$	M1	
	$t=\frac{292.4}{u}$	A1	
	$200\sin30=\frac{1}{2}(-9.8)(\frac{292.4}{u})^2+u\sin50(\frac{292.4}{u})$	M1	
	$u^2=2693$	A1	
	$u=51.9$	A1	
(b)	**Alternative:**		
	$0=(u\sin30)^2-2g\cos20.s$	M1	
	$s=\frac{(51.9\sin30)^2}{2(9.8)\cos20}$	m1A1	
	$s=36.6$	A1	

Question 7: June 2012 – Q5

(a)	$0=15t\sin30-\frac{1}{2}g\cos25t^2$	M1A1	
	$t=\frac{15\sin30}{\frac{1}{2}g\cos25}$	M1	
	$t=1.69$ sec.	A1F	4
(b)	$\perp$ to plane $\dot{y}=15\sin30-g\cos25\times\frac{15\sin30}{\frac{1}{2}g\cos25}$	M1	
	$\dot{y}=-7.5$ ms^{-1}	A1F	
	$\parallel$ to plane $\dot{x}=15\cos30-g\sin25\times\frac{15\sin30}{\frac{1}{2}g\cos25}$	M1	
	$\dot{x}=5.995766$ or 6.00 ms^{-1}	A1F	
	Restitution: Rebound $\dot{y}=\frac{2}{3}\times7.5=5$ ms^{-1}	M1	
	$\dot{x}$ unchanged	B1	
	Speed of rebound $=\sqrt{5.995766^2+5^2}$ $=7.81$ ms^{-1}	m1 A1F	8
	Total		**12**