Polynomials

Algebraic division

We use algebraic division to divide algebraic expression:

$(x^3+3x^2-5x+2)\div(x+2)$

$$
\begin{array}{r}
x^2+x-7 \\
x+2\,\overline{)\,x^3+3x^2-5x+2} \\
-\underline{x^3+2x^2} \qquad\qquad \\
x^2-5x \qquad \\
-\underline{x^2+2x} \qquad \\
-7x+2 \\
-\underline{-7x-14} \\
16
\end{array}
$$

Conclusion: $x^3+3x^2-5x+2=(x+2)(x^2+x-7)+16$

The remainder and factor theorem.

a) The remainder theorem.

> The remainder of the division of $f(x)$ by $(x-a)$ is $f(a)$.

Example: $f(x)=x^3+2x-5x+3$

- If we divide $f(x)$ by $(x-2)$, the remainder is $f(2)=(2)^3-2\times2-5\times2+3=-3$
- If we divide $f(x)$ by $(2x-3)$, the remainder is $f(\frac{3}{2})=(\frac{3}{2})^3-2\times\frac{3}{2}-5\times\frac{3}{2}+3=-\frac{33}{8}$

> The remainder of the division of $f(x)$ by $(ax-b)$ is $f\left(\frac{b}{a}\right)$.

b) The factor theorem.

> The following statements are equivalent:
> - a is a root of f
> - $f(a)=0$ (The remainder of the division by $(x-a)$ is 0)
> - $(x-a)$ is a factor of $f(x)$.

Example: $f(x)=x^3+2x-6x+3$

Show that $(x-1)$ is a factor of f.

$f(1)=1^3+2\times1-6\times1+3=1+2-6+3=0$

1 is a root of f , $(x-1)$ is a factor of f.

Factorising cubic expressions

To factorise a cubic expression, $f(x)$,

1) you need to find or be given a factor or a root of f, *for example* "a".
2) Use the algebraic division to factorise f by $(x-a)$

$$f(x)=(x-a)(bx^2+cx+d)$$

3) Factorise the quadratic expression bx^2+cx+d.

Example: a) *Show that 2 is a root of* $f(x)=2x^3+3x^2-11x-6$

b) *Factorise fully* $f(x)$.

a) $f(2)=2\times2^3+3\times2^2-11\times2-6=16+12-22-6=0$ $(x-2)$ *is a factor*

b) $f(x)=(x-2)(2x^2+7x+3)=(x-2)(2x+1)(x+3)$.