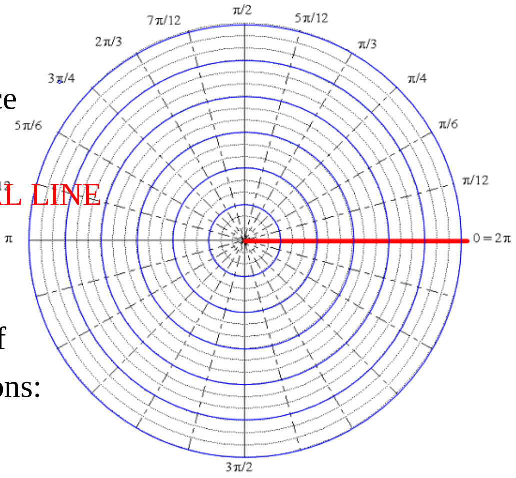
Polar coordinates

Definitions

A point M can be placed in a set of two axis using CARTESIAN coordinates $M(x, y)$



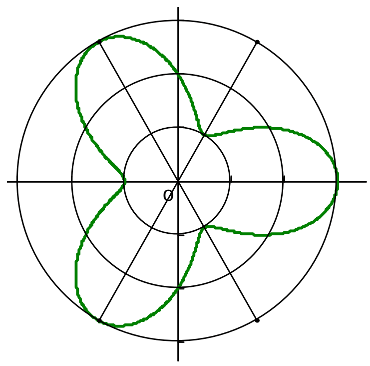
This position can also be determied by the distance from the origin O or POLE and the angle made by the line OM with the positive x-axis or INITIAL LINE

The POLAR coordiantes of M(r, θ)

In order to have unicity in the polar coordinates of a given point, we will use the following conventions:

$r > 0 \;\; and \;\; -\pi < \theta \le \pi \; or \; 0 \le \theta < 2\pi$

Curve in Polar coordinates



In cartesian coordinates, an explicit equation of a curve will be given as $y = f(x)$.

In polar coordinates, an explicit equation of a curve will be given as $r = f(\theta)$.

Examples : $r = 2Sin\theta$, $r = e^{-2\theta}$, $r = 3, \ldots$

Conversions

A point M has cartesian coordinates $M(x, y)$ and polar coordinates M(r, θ)

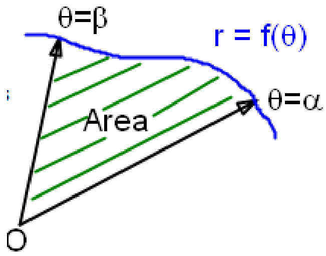
Using the pythagoras' theorem and trigonometry, we have

$$\begin{cases} x = rCos\theta \\ y = rSin\theta \end{cases} \quad and \quad \begin{cases} r^2 = x^2 + y^2 \\ Tan\theta = \dfrac{y}{x} \end{cases}$$

Note : If $x > 0, \theta = ArcTan(\frac{y}{x})$ but if $x < 0, \theta = ArcTan(\frac{y}{x}) \pm \pi$

Area bounded by a polar curve

The are bounded by a polar curve and the ray $\theta=\alpha$ and $\theta=\beta$



is $$A = \frac{1}{2}\int_{\alpha}^{\beta} r^2 d\theta$$

IMPORTANT : This formula is valid only if $r > 0$ *for* $\alpha \le \theta \le \beta$.