Polar coordinates – Exam questions

Question: Jan 2007 Q2

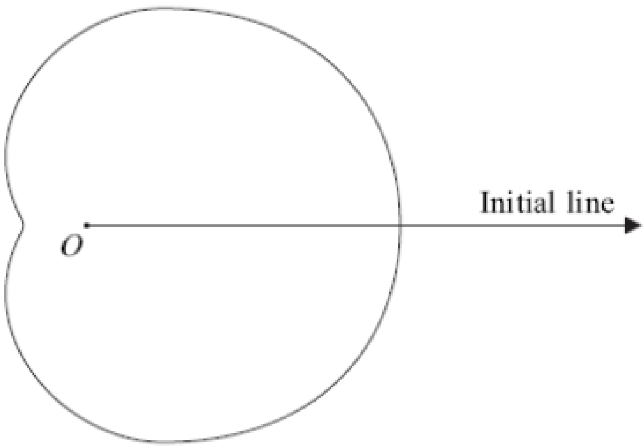
A curve has polar equation $r(1 - \sin\theta) = 4$. Find its cartesian equation in the form $y = f(x)$. *(6 marks)*

Question: Jan 2007 Q7

A curve C has polar equation

$$r = 6 + 4\cos\theta, \qquad -\pi \leqslant \theta \leqslant \pi$$

The diagram shows a sketch of the curve C, the pole O and the initial line.



(a) Calculate the area of the region bounded by the curve C. *(6 marks)*

(b) The point P is the point on the curve C for which $\theta = \frac{2\pi}{3}$.

The point Q is the point on C for which $\theta = \pi$.

Show that QP is parallel to the line $\theta = \frac{\pi}{2}$. *(4 marks)*

(c) The line PQ intersects the curve C again at a point R.

The line RO intersects C again at a point S.

(i) Find, in surd form, the length of PS. *(4 marks)*

(ii) Show that the angle OPS is a right angle. *(1 mark)*

Question: June 2008 Q3

(a) Show that $x^2 = 1 - 2y$ can be written in the form $x^2 + y^2 = (1 - y)^2$. *(1 mark)*

(b) A curve has cartesian equation $x^2 = 1 - 2y$.

Find its polar equation in the form $r = f(\theta)$, given that $r > 0$. *(5 marks)*

Question: June 2008 Q8

The polar equation of a curve C is

$$r = 5 + 2\cos\theta, \qquad -\pi \leqslant \theta \leqslant \pi$$

(a) Verify that the points A and B, with **polar coordinates** $(7, 0)$ and $(3, \pi)$ respectively, lie on the curve C. *(2 marks)*

(b) Sketch the curve C. *(2 marks)*

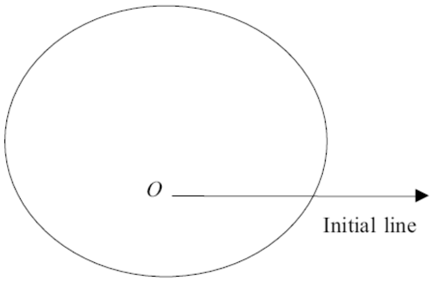
(c) Find the area of the region bounded by the curve C. *(6 marks)*

(d) The point P is the point on the curve C for which $\theta = \alpha$, where $0 < \alpha \leqslant \frac{\pi}{2}$. The point Q lies on the curve such that POQ is a straight line, where the point O is the pole. Find, in terms of α, the area of triangle OQB. *(4 marks)*

Question: Jan 2006 Q6

(a) A circle C_1 has cartesian equation $x^2 + (y-6)^2 = 36$. Show that the polar equation of C_1 is $r = 12\sin\theta$. *(4 marks)*

(b) A curve C_2 with polar equation $r = 2\sin\theta + 5$, $0 \leqslant \theta \leqslant 2\pi$ is shown in the diagram.

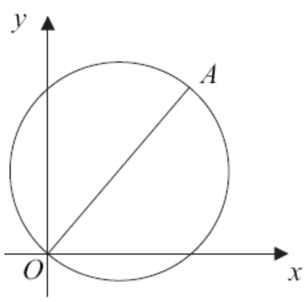


Calculate the area bounded by C_2. *(6 marks)*

(c) The circle C_1 intersects the curve C_2 at the points P and Q. Find, in surd form, the area of the quadrilateral $OPMQ$, where M is the centre of the circle and O is the pole. *(6 marks)*

Question: June 2009 Q3

The diagram shows a sketch of a circle which passes through the origin O.



The equation of the circle is $(x-3)^2 + (y-4)^2 = 25$ and OA is a diameter.

(a) Find the cartesian coordinates of the point A. *(2 marks)*

(b) Using O as the pole and the positive x-axis as the initial line, the polar coordinates of A are (k, α).

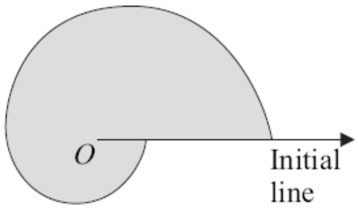
(i) Find the value of k and the value of $\tan\alpha$. *(2 marks)*

(ii) Find the polar equation of the circle $(x-3)^2 + (y-4)^2 = 25$, giving your answer in the form $r = p\cos\theta + q\sin\theta$. *(4 marks)*

Question: June 2009 Q7

The diagram shows the curve C_1 with polar equation

$$r = 1 + 6e^{-\frac{\theta}{\pi}}, \quad 0 \leqslant \theta \leqslant 2\pi$$



(a) Find, in terms of π and e, the area of the shaded region bounded by C_1 and the initial line. *(5 marks)*

(b) The polar equation of a curve C_2 is

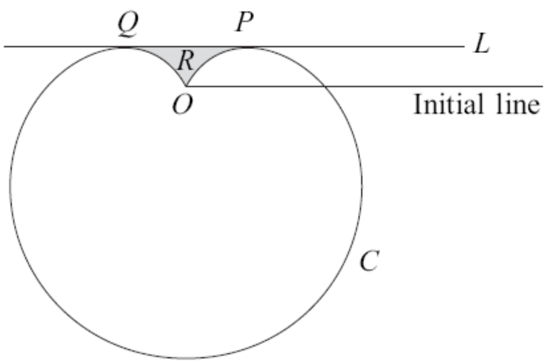
$$r = e^{\frac{\theta}{\pi}}, \quad 0 \leqslant \theta \leqslant 2\pi$$

Sketch the curve C_2 and state the polar coordinates of the end-points of this curve. *(4 marks)*

(c) The curves C_1 and C_2 intersect at the point P. Find the polar coordinates of P. *(5 marks)*

Question: Jan 2010 Q8

The diagram shows a sketch of a curve C and a line L, which is parallel to the initial line and touches the curve at the points P and Q.



The polar equation of the curve C is

$$r = 4(1 - \sin\theta), \qquad 0 \leqslant \theta < 2\pi$$

and the polar equation of the line L is

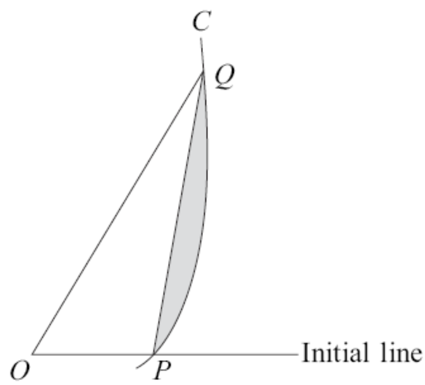
$$r\sin\theta = 1$$

(a) Show that the polar coordinates of P are $\left(2, \frac{\pi}{6}\right)$ and find the polar coordinates of Q. *(5 marks)*

(b) Find the area of the shaded region R bounded by the line L and the curve C. Give your answer in the form $m\sqrt{3} + n\pi$, where m and n are integers. *(11 marks)*

Question: Jan 2008 Q2

The diagram shows a sketch of part of the curve C whose polar equation is $r = 1 + \tan\theta$. The point O is the pole.



The points P and Q on the curve are given by $\theta = 0$ and $\theta = \frac{\pi}{3}$ respectively.

(a) Show that the area of the region bounded by the curve C and the lines OP and OQ is

$$\frac{1}{2}\sqrt{3} + \ln 2$$ *(6 marks)*

(b) Hence find the area of the shaded region bounded by the line PQ and the arc PQ of C. *(3 marks)*

Question: Jan 2008 Q6

A curve C has polar equation

$$r^2 \sin 2\theta = 8$$

(a) Find the cartesian equation of C in the form $y = \mathrm{f}(x)$. *(3 marks)*

(b) Sketch the curve C. *(1 mark)*

(c) The line with polar equation $r = 2\sec\theta$ intersects C at the point A. Find the polar coordinates of A. *(4 marks)*

Question: June 2007 Q4

(a) Show that $(\cos\theta + \sin\theta)^2 = 1 + \sin 2\theta$. *(1 mark)*

(b) A curve has cartesian equation

$$(x^2 + y^2)^3 = (x + y)^4$$

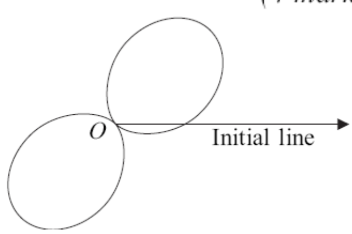
Given that $r \geqslant 0$, show that the polar equation of the curve is

$$r = 1 + \sin 2\theta$$ *(4 marks)*

(c) The curve with polar equation

$$r = 1 + \sin 2\theta, \quad -\pi \leqslant \theta \leqslant \pi$$

is shown in the diagram.



(i) Find the two values of θ for which $r = 0$. *(3 marks)*

(ii) Find the area of one of the loops. *(6 marks)*

Polar coordinates – Exam questions

Question: Jan 2007 Q2

$r - r\sin\theta = 4$	M1	
$r - y = 4$	B1	
$r = y + 4$	A1	
$x^2 + y^2 = (y+4)^2$	M1	
$x^2 + y^2 = y^2 + 8y + 16$	A1F	
$y = \dfrac{x^2 - 16}{8}$	A1	6
Total		**6**

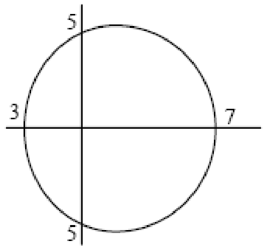
Question: Jan 2007 Q7

(a)	$\text{Area} = \frac{1}{2}\int (6 + 4\cos\theta)^2 \, d\theta$	M1	
	$= \frac{1}{2}\left(\int_{-\pi}^{\pi} 36 + 48\cos\theta + 16\cos^2\theta\right) d\theta$	B1 B1	
	$= \left(\int_{-\pi}^{\pi} 18 + 24\cos\theta + 4(\cos 2\theta + 1)\right) d\theta$	M1	
	$= \left[22\theta + 24\sin\theta + 2\sin 2\theta\right]_{-\pi}^{\pi}$	A1F	
	$= 44\pi$	A1	6
(b)	At P, $r = 4$; At Q, $r = 2$;	B1	
	P {$x =$} $r\cos\theta = 4\cos\frac{2\pi}{3} = -2$	M1	
	Q {$x =$} $r\cos\theta = 2\cos\pi = -2$	A1	
	Since P and Q have same 'x', PQ is vertical so QP is parallel to the vertical line $\theta = \frac{\pi}{2}$	E1	4
(i)	$OP = 4$; $OS = 8$;	B1	
	Angle $POS = \frac{\pi}{3}$	B1	
	$PS^2 = 4^2 + 8^2 - 2\times 4\times 8\times\cos\frac{\pi}{3}$ oe	M1	
	$PS = \sqrt{48}$ $\{= 4\sqrt{3}\}$	A1	4
(ii)	Since $8^2 = 4^2 + \left(\sqrt{48}\right)^2$, $OS^2 = OP^2 + PS^2 \Rightarrow OPS$ is a right angle. (Converse of Pythagoras Theorem)	E1	1
	Total		**15**

Question: June 2008 Q3

(a)	$x^2 + y^2 = 1 - 2y + y^2 \Rightarrow x^2 + y^2 = (1-y)^2$	B1	1
(b)	$x^2 + y^2 = r^2$	M1	
	$y = r\sin\theta$	M1	
	$x^2 = 1 - 2y$ so $x^2 + y^2 = (1-y)^2$ $\Rightarrow r^2 = (1 - r\sin\theta)^2$	A1	
	$r = 1 - r\sin\theta$ or $r = -(1 - r\sin\theta)$ $r(1+\sin\theta) = 1$ or $r(1-\sin\theta) = -1$	m1	
	$r > 0$ so $r = \dfrac{1}{1+\sin\theta}$	A1	5
	Total		**6**

Question: June 2008 Q8

(a)	$\theta = 0$, $r = 5 + 2\cos 0 = 7$ {A lies on C}	B1	
	$\theta = \pi$, $r = 5 + 2\cos\pi = 3$ {B lies on C}	B1	2
(b)		B1 B1	2
(c)	$\text{Area} = \frac{1}{2}\int (5 + 2\cos\theta)^2 d\theta$	M1	
	$= \frac{1}{2}\int_{-\pi}^{\pi}\left(25 + 20\cos\theta + 4\cos^2\theta\right) d\theta$	B1 B1	
	$= \frac{1}{2}\int_{-\pi}^{\pi}\left(25 + 20\cos\theta + 2(\cos 2\theta + 1)\right) d\theta$	M1	
	$= \frac{1}{2}\left[27\theta + 20\sin\theta + \sin 2\theta\right]_{-\pi}^{\pi}$	A1F	
	$= 27\pi$	A1	6
(d)	Triangle OBQ with $OB = 3$ and angle $BOQ = \alpha$	B1	
	$OQ = 5 + 2\cos(-\pi + \alpha)$	M1	
	Area of triangle $OQB = \frac{1}{2} OB \times OQ \sin\alpha$	m1	
	$= \frac{3}{2}(5 - 2\cos\alpha)\sin\alpha$	A1	4
	Total		**14**

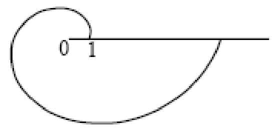
Question: Jan 2006 Q6

(a)	$x^2+y^2-12y+36=36$	M1 M1 m1	
	$r^2-12r\sin\theta+36=36$		
	$\Rightarrow r=12\sin\theta$	A1	4
(b)	Area $=\frac{1}{2}\int(2\sin\theta+5)^2\ d\theta$.	M1	
	$..=\frac{1}{2}\int_0^{2\pi}(4\sin^2\theta+20\sin\theta+25)d\theta$	B1 B1	
	$=\frac{1}{2}\int_0^{2\pi}(2(1-\cos 2\theta)+20\sin\theta+25)d\theta$	M1	
	$=\frac{1}{2}[27\theta-\sin 2\theta-20\cos\theta]_0^{2\pi}$	A1√	
	$=27\pi$.	A1	6
(c)	At intersection $12\sin\theta=2\sin\theta+5$	M1	
	$\Rightarrow \sin\theta=\frac{5}{10}$	A1	
	Points $\left(6,\frac{\pi}{6}\right)$ and $\left(6,\frac{5\pi}{6}\right)$	A1	
	OPMQ is a rhombus of side 6		
	Area $=6\times6\times\sin\frac{2\pi}{3}$ oe	M1 A1	
	$=18\sqrt{3}$	A1	6
	Total		**16**

Question: June 2009 Q3

(a)	Centre of circle is $M(3, 4)$	B1	
	$A(6, 8)$	B1	2
(i)	$k=OA=10$	B1	
	$\tan\alpha=\frac{y_A}{x_A}=\frac{4}{3}$	B1	2
(ii)	$x^2+y^2-6x-8y+25=25$	B1	
	$r^2-6r\cos\theta-8r\sin\theta=0$	M1M1	
	{$r=0$, origin} Circle: $r=6\cos\theta+8\sin\theta$	A1	4
	ALTn Circle has eqn $r=OA\cos(\alpha-\theta)$ $r=OA\cos\alpha\cos\theta+OA\sin\alpha\sin\theta$ Circle: $r=6\cos\theta+8\sin\theta$	(M2) (m1) (A1)	
	Total		**8**

Question: June 2009 Q7

(a)	Area $=\frac{1}{2}\int\left(1+6e^{-\frac{\theta}{\pi}}\right)^2\ d\theta$	M1	
	$=\frac{1}{2}\int_0^{2\pi}\left(1+12e^{-\frac{\theta}{\pi}}+36e^{-\frac{2\theta}{\pi}}\right)d\theta$	B1	
		B1	
	$=\frac{1}{2}\left[\theta-12\pi e^{-\frac{\theta}{\pi}}-18\pi e^{-\frac{2\theta}{\pi}}\right]_0^{2\pi}$	m1	
	$=\pi(16-6e^{-2}-9e^{-4})$	A1	5
(b)		B1 B1	
	End-points $(1, 0)$ and $(e^2, 2\pi)$	B2,1,0	4
(c)	$e^{\frac{\theta}{\pi}}=1+6e^{-\frac{\theta}{\pi}}$	M1	
	$\left(e^{\frac{\theta}{\pi}}\right)^2-e^{\frac{\theta}{\pi}}-6=0$	m1	
	$\left(e^{\frac{\theta}{\pi}}-3\right)\left(e^{\frac{\theta}{\pi}}+2\right)=0$	m1	
	$e^{\frac{\theta}{\pi}}>0$ so $e^{\frac{\theta}{\pi}}=3$	E1	
	Polar coordinates of *P* are $(3, \pi\ln 3)$	A1	5
	Total		**14**

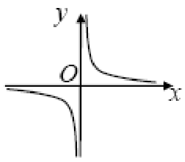
Question: Jan 2010 Q8

a)	$4\sin\theta(1-\sin\theta)=1$	M1	
	$4\sin^2\theta-4\sin\theta+1=0$	A1	
	$(2\sin\theta-1)^2=0 \Rightarrow \sin\theta=0.5$	m1	
	$\theta=\frac{\pi}{6},\quad \theta=\frac{5\pi}{6},\ r=2$	A2,1	
	$[P\left(2,\ \frac{\pi}{6}\right)\quad Q\left(2,\ \frac{5\pi}{6}\right)]$		5
b)	Area triangle OPQ $=\frac{1}{2}\times 2\times r_Q\times\sin POQ$	M1	
	Angle $POQ=\frac{5\pi}{6}-\frac{\pi}{6}\quad\left(=\frac{2\pi}{3}\right)$	m1	
	Area triangle $OPQ=2\sin\frac{2\pi}{3}=\sqrt{3}$	A1	
	Unshaded area bounded by line OP and arc $OP=\frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}[4(1-\sin\theta)]^2\,d\theta$	M1	
	$=8\int(1-2\sin\theta+\sin^2\theta)\,d\theta$	B1	
	$=8\int\left(1-2\sin\theta+\frac{1-\cos2\theta}{2}\right)d\theta$	M1	
	$=8\left[\theta+2\cos\theta+\frac{\theta}{2}-\frac{\sin2\theta}{4}\right]\ (+c)$	A1F	
	$8\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}(1-\sin\theta)^2\,d\theta=$		
	$8\times\left[\frac{3\theta}{2}+2\cos\theta-\frac{\sin2\theta}{4}\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}}$		
	$=8\times\{\frac{3\pi}{4}-\left(\frac{3\pi}{12}+2\cos\frac{\pi}{6}-\frac{1}{4}\sin\frac{2\pi}{6}\right)\}$	m1	
	$=8\times\left(\frac{\pi}{2}-\sqrt{3}+\frac{\sqrt{3}}{8}\right)\quad\{=4\pi-7\sqrt{3}\}$	A1F	
	Shaded area = Area of triangle OPQ − $2\times\frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}[4(1-\sin\theta)]^2\,d\theta$	M1	
	Shaded area = $\sqrt{3}-16\left(\frac{\pi}{2}-\sqrt{3}+\frac{\sqrt{3}}{8}\right)=15\sqrt{3}-8\pi$	A1	11
	Total		**16**

Question: Jan 2008 Q2

(a)	Area $=\frac{1}{2}\int(1+\tan\theta)^2\,d\theta$	M1	
	$\dots=\frac{1}{2}\int(1+2\tan\theta+\tan^2\theta)\,d\theta$	B1	
	$=\frac{1}{2}\int(\sec^2\theta+2\tan\theta)\,d\theta$	M1	
	$=\frac{1}{2}[\tan\theta+2\ln(\sec\theta)]_0^{\frac{\pi}{3}}$	A1√ B1√	
	$=\frac{1}{2}[(\sqrt{3}+2\ln2)-0]=\frac{\sqrt{3}}{2}+\ln2$	A1	6
(b)	$OP=1;\ OQ=1+\tan\frac{\pi}{3}$	B1	
	Shaded area = 'answer (a)' $-\frac{1}{2}OP\times OQ\times\sin\left(\frac{\pi}{3}\right)$	M1	
	$=\frac{\sqrt{3}}{2}+\ln2-\frac{\sqrt{3}}{4}(1+\sqrt{3})$	A1	3
	$=\frac{\sqrt{3}}{4}+\ln2-\frac{3}{4}$		
	Total		**9**

Question: Jan 2008 Q6

(a)	$r^2\,2\sin\theta\cos\theta=8$	M1	
	$x=r\cos\theta\quad y=r\sin\theta$	M1	
	$xy=4\ ,\quad y=\frac{4}{x}$	A1	3
(b)		B1	1
(c)	$r=2\sec\theta$ is $x=2$	B1	
	Sub $x=2$ in $xy=4\ \Rightarrow 2y=4$	M1	
	In cartesian, $A(2,2)$		
	$\Rightarrow\tan\theta=\frac{y}{x}=1\ \Rightarrow\ \theta=\frac{\pi}{4}$		
	$\Rightarrow r=\sqrt{x^2+y^2}=\sqrt{8}$	M1	
	$\theta=\frac{\pi}{4}\ ;\ r=\sqrt{8}$	A1	4
	Altn2: Eliminating r to reach eqn. in $\cos\theta$ and $\sin\theta$ only (M1) $\theta=\frac{\pi}{4}$ (A1)		
	Substitution $r=2\sec\left(\frac{\pi}{4}\right)$ (m1)		
	$r=\sqrt{8}$ (A1) OE surd		
	Total		**8**

Question: June 2007 Q4

(a)	$(\cos\theta+\sin\theta)^2 = \cos^2\theta+\sin^2\theta+2\cos\theta\sin\theta$ $=1+\sin 2\theta$	B1	1
(b)	$(x^2+y^2)^3=(x+y)^4$		
	$(r^2)^3=(r\cos\theta+r\sin\theta)^4$	M2,1,0	
	$r^6=r^4(\cos\theta+\sin\theta)^4$		
	$r^6=r^4(1+\sin 2\theta)^2$	M1	
	$r^2=(1+\sin 2\theta)^2$		
	$\Rightarrow r=(1+\sin 2\theta)\ \{r\geq 0\}$	A1	4
(i)	$r=0\Rightarrow \sin 2\theta=-1$		
	$2\theta=\sin^{-1}(-1);\ =-\frac{\pi}{2},\frac{3\pi}{2}$	M1	
	$\theta=-\frac{\pi}{4};\ \frac{3\pi}{4}$	A1A1ft	3
(ii)	Area $=\frac{1}{2}\int(1+\sin 2\theta)^2\,d\theta$	M1	
	$=\frac{1}{2}\int(1+2\sin 2\theta+\sin^2 2\theta)\,d\theta$	B1	
	$=\frac{1}{2}\int\left(1+2\sin 2\theta+\frac{1}{2}(1-\cos 4\theta)\right)d\theta$	M1	
	$=\left[\frac{3}{4}\theta-\frac{1}{2}\cos 2\theta-\frac{1}{16}\sin 4\theta\right]$	A1ft	
	$=\left[\frac{3}{4}\theta-\frac{1}{2}\cos 2\theta-\frac{1}{16}\sin 4\theta\right]_{-\frac{\pi}{4}}^{\frac{3\pi}{4}}$		
	$=\left(\frac{9\pi}{16}\right)-\left(-\frac{3\pi}{16}\right)$	m1	
	$=\frac{3\pi}{4}$	A1	6
	Total		**14**