Limits

Limits you have to know:

You are allowed to use the following results without proof:

• when $x \to \infty$, $x^k e^{-x} \to 0$ for any real number k.

• when $x \to 0$, $x^k \ln(x) \to 0$ for $k > 0$.

Improper integrals

The integral $\int_a^b f(x)dx$ is said IMPROPER if

a) the interval of integration is infinite,

or b) $f(x)$ is not defined at one or both of the end points $x = a$ *and* $x = b$.

Method

To work out if an improper integral has a value or not (exists or not)

1) Replace "∞" or "a", the value where f is not defined, by a letter. "N" for example.
2) Integrate to find an expression in terms of "N".
3) Work out the limit of this expression when "N" tends to "∞" or "a".
4) If the limit exists then the improper integral has a value. If the limit is "∞", the improper integral does not exist.

Example: $\int_0^{\infty} \frac{1}{1+x^2} dx$ is an improper integral.

Let's work out $\int_0^{N} \frac{1}{1+x^2} dx = \left[\text{Arc}\tan(x)\right]_0^N = \text{Arctan(N)} - \text{Arctan(0)}$

$\text{Arctan}(0) = 0$ and when $\text{N} \to \infty, \text{Arctan(N)} \to \frac{\pi}{2}$.

conclusion: $\int_0^{\infty} \frac{1}{1+x^2} dx$ *exists* and $\int_0^{\infty} \frac{1}{1+x^2} dx = \frac{\pi}{2}$