# Integration

## Indefinite integrals

Integration is the "opposite" of differentiation.

If $y = f(x)$ is a given function, to integrate $f$ means finding a function $F(x)$

so that $\frac{dF}{dx} = f$

$F$ is called an INTEGRAL of $f$ and it is noted $\int f(x)dx$

*Note*: An integral is not unique. If $F(x)$ is an integral, then $F(x) + c$ is also one.

$$\int f(x)dx = F(x) + c \qquad \textit{where } c \textit{ is a constant}$$

## Integrating $x^n$

The formula tells you how to integrate *powers of* $x$.

$$\int x^n dx = \frac{1}{n+1}x^{n+1} + c \qquad \textit{for all } n \neq -1$$

## Rules of integrations

$f(x)$ *and* $g(x)$ are two functions , $a$ is a constant

$$\int a \times f(x)dx = a \times \int f(x)dx$$

$$\int f(x) + g(x)dx = \int f(x)dx + \int g(x)dx$$

*Examples*: $\int x^3 dx = \frac{1}{4}x^4 + c$ , $\int (3x^2 - 3x)\,dx = 3 \times \frac{1}{3}x^3 - 3 \times \frac{1}{2}x^2 + c = x^3 - \frac{3}{2}x^2 + c$

## Integrating to find the equation of a curve

A curve $y = f(x)$ is going through the point $A(x_A, y_A)$ and $\frac{dy}{dx} = f'(x)$ *is given.*

To find the equation of the curve,

- integrate $f'(x)$ : $\int f'(x)dx = F(x) + c$
- find the value of the constant $c$ using the coordinates of A.

Example: The curve $y = f(x)$ goes through A(2,9) and $\frac{dy}{dx} = 3x^2$.

Find the eqaution of the curve.

- $\int 3x^2 dx = x^3 + c$ *so* $y = x^3 + c$
- $A(2,9)$ belongs to the curve so $9 = (2)^3 + c$ $c = 1$
- the eqaution of the curve is $y = x^3 + 1$