Friction

Consider a particle of mass M, which is on a rough horizontal plane. Given a horizontal force of magnitude S is applied (as in Figure 1), then assuming the particle remains in equilibrium, the magnitude of the frictional force, F, opposing any motion, will be equal to S, i.e. $F = S$.

If S is gradually increased, then F also increases, as long as the particle remains at rest, so that the equation $F = S$ still holds true. **But**, F cannot increase indefinitely, it can only increase up to a limit F_{MAX}.

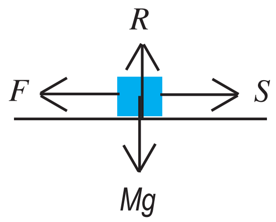

Figure 1

How large the frictional force, F, can become is determined by
1. The force between the surfaces in contact *and*
2. The types of surfaces

The normal reaction, is the vertical force perpendicular to the contact surface. It can be shown that F_{MAX} is proportional to the magnitude of the normal reaction, R.
Consequently,

$$\boxed{F_{MAX} = \mu \times R}$$

where μ, known as the Coefficient of (Static) Friction, is a constant, which depends on the roughness of the surface. Slipping will occur if S is increased further.
Note:
1. The frictional force is said to be limiting when it equals its maximum, F_{MAX}
2. The inequality $F \leq \mu R$ is always true
3. A smooth plane gives $\mu = 0$, which means $F_{MAX} = 0$
4. When there is motion, friction is slightly smaller than limiting friction, but unless otherwise informed the assumption that friction $= F_{MAX}$ will be adopted.

Worked Example 1.

A horizontal force of 20 N acts on a particle of mass 7 kg on a rough horizontal plane. Given the particle is on the point of slipping, what is the coefficient of friction, between the particle and the plane?

Solution
Resolving Vertically:

$$R = mg = 68.67 \text{ N}$$

Resolving Horizontally:

$$20 - F = 0, \Rightarrow F = 20 \text{ N}$$

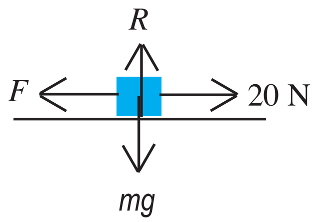

Figure 2

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As the particle is on the point of slipping, fiction is limiting, $F = F_{MAX}$, so:

$$\begin{aligned} F_{MAX} &= \mu R \\ 20 &= 68.67\mu \\ \Rightarrow \mu &= \frac{20}{68.67} = 0.29 \quad (2 \text{ s.f.}) \end{aligned}$$

Worked Example 2.

The coefficient of friction between a particle, of mass 8 kg, and a rough horizontal plane is 0.4. Given a horizontal force of 29 N acts on the particle (as in Figure 3) does slipping occur?

Solution

Resolving Vertically:

$$R = mg = 78.48 \text{ N}$$

Also: $\quad F_{MAX} = 0.4 \times 78.48 = 31 \text{ N} \quad (2 \text{ s.f.})$

And for equilibrium:

$F = 29 < 31 = F_{MAX}$, so no motion will occur.

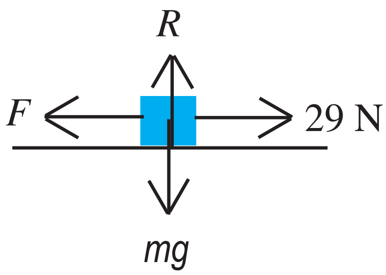

Figure 3

Exercises

1. A horizontal force of 15 N acts on a particle of mass 14 kg on a rough horizontal plane. Given the particle is on the point of sliding, what is the coefficient of friction between the particle and the plane?

2. A horizontal force of $\frac{3}{2}g$ N acts on a particle of mass 9 kg on a rough horizontal plane. Given the particle is on the point of sliding, what is the coefficient of friction between the particle and the plane?

3. The coefficient of friction between a particle, of mass 9.5 kg, and a rough horizontal plane is 0.12. Given that a horizontal force of 12 N acts on the particle, does slipping occur?

4. The coefficient of friction between a particle, of mass 6 kg, and a rough horizontal plane is $\frac{1}{3}$. Given that a horizontal force of $2g$ N acts on the particle, does slipping occur?

5. A horizontal force of T N acts on a particle of mass 12 kg, which is on a rough horizontal plane. Given that the particle is on the point of slipping and the coefficient of friction is 0.35, what is T?

6. The coefficient of friction between a particle, of mass M kg, and a rough horizontal plane is μ. A horizontal force of $\frac{1}{5}R$ N, where R is the normal contact force, acts on the particle. Given the particle is on the point of slipping what is the value of μ?

Answers (all to 2 s.f.)

1. 0.11 2. $\frac{3}{18} = \frac{1}{6} \approx 0.17$ 3. Yes; slipping occurs; horizontal force $= 12 > 11$ (F_{MAX})
4. No; slipping does not occur; horizontal force $= 2g \leq 2g$ (F_{MAX}) 5. 41 N 6. 0.20

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