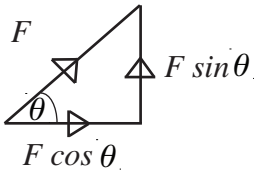
Forces acting at an angle: Resolving Forces

A force that acts at an angle can be split into two perpendicular components.



Newton's Second Law can be applied in each of the resolved directions.

Worked Example 1.

A computer base unit of mass 6 kg is dragged along a smooth desk. If the tension in each arm of the person dragging it is 20 N and it acts at 25° above the horizontal, what is the acceleration of the base unit and what is its normal reaction?

Solution

Figure 1 shows the forces acting on the base unit. Firstly the acceleration, a, needs to be calculated.

The resultant horizontal force is $40\cos 25°$.
Using Newton's Second Law of Motion:

$$\begin{aligned} F &= ma \\ 40\cos 25° &= 6 \times a \\ \Rightarrow a &= 6.0 \text{ m s}^{-2} (2 \text{ s.f.}) \end{aligned}$$

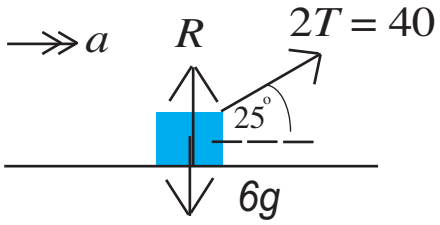


Figure 1

In order to calculate the normal reaction force, resolve vertically:

$$\begin{aligned} R + 40\sin 25° - 6 \times 9.81 &= 0 \\ \Rightarrow R = 58.56 - 16.90 &= 42 \text{ N (2 s.f.)} \end{aligned}$$

Worked Example 2.

Two tug boats are towing a large boat, of mass 13750 kg, back to shore. Tug boat 1 is pulling with a force of $T_1 = 7500$ N at an angle of 30° north of the forward motion (see Figure 2) and tug boat 2 is pulling with a force of $T_2 = 8500$ N at an angle θ south of the forward motion. If there is a resistive motion of 1050 N opposing the eastern motion, what is the acceleration of the large boat?

Solution

Firstly, calculate the unknown angle θ.

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Resolving perpendicular to the direction of motion gives:

$$\begin{aligned} T_1 \sin 30° - T_2 \sin\theta &= 0 \\ \frac{7500(\frac{1}{2})}{8500} &= \sin\theta \\ \Rightarrow \theta &= 26° \text{ (2 s.f.)} \end{aligned}$$

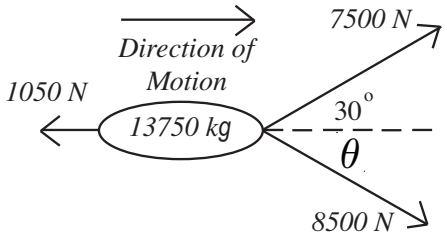

Figure 2 (View from above)

In order to calculate the acceleration, resolve in the direction of motion, which gives the resultant force as $7500 \cos 30° + 8500 \cos 26° - 1050$.
Applying Newton's Second Law gives:

$$\begin{aligned} 7500\cos 30° + 8500 \cos 26° - 1050 &= 13750a \\ \Rightarrow a &= 0.95 \text{ m s}^{-2} \text{ (2 s.f.)} \end{aligned}$$

Exercises

1. A computer base unit of mass 4.5 kg is dragged along a smooth desk. If the tension in each arm of the person dragging it is 16 N and acts at 22° above the horizontal, then what is the normal reaction force?

2. A computer base unit of mass 7.5 kg is dragged along a smooth desk. If the normal contact force is 23 N and the tension in the arm of the person dragging it acts at 23° to the horizontal, then what is the total tension in the person's arms?

3. Two tug boats are towing a large boat, of mass 22500 kg, back to shore. Tug boat 1 is pulling with a force of 5500 N at an angle of 35° north of the forward motion (similar to Worked Example 2) and tug boat 2 is pulling with a force of 4907.8 N at an angle 40° south of the forward motion. If the large boat is being pulled with constant velocity, and there is a resistive force to the motion, then what size is the resistive force?

4. Two tug boats are towing a large boat, of mass M kg, back to shore. Tug boat 1 is pulling with a force of T_1 N at an angle of 25° north of the forward motion (like in Worked Example 2) and tug boat 2 is pulling with a force of T_2 N at an angle of 25° south of the forward motion. If the large boat is being pulled with constant velocity, and there is a resistive force of 4000 N to the motion, then what are the magnitudes of the two forces T_1 and T_2?

5. A child on a sledge is being pulled along a horizontal path by its parent. The child and sledge have a combined mass of 20 kg and there is a normal contact force of 124.5 N. Given there is no resistance to motion and the parent pulls with a force of 125 N at an angle θ to the horizontal, then what is the angle θ?

6. A child on a sledge is being pulled along a horizontal path by its parent. The child and sledge have a combined mass of 18 kg and there is a normal contact force of 135 N. Given there is no resistance to motion and the parent pulls with a force of F N at an angle 25° to the horizontal, then what is F?

Answers (all to 2 s.f.)

1. 32 N 2. 130 N 3. 8300 N 4. $F_1 = F_2 = 2200$ N 5. 35° 6. 98 N

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