

Displacement-time and Velocity-time graphs

In leaflet 1.9 several constant acceleration formulae were introduced. However, graphs can often be used to describe mathematical models. In this case two types of graph are commonly used.

1. Displacement-time graphs.

Any point on such a graph has coordinates (t,s) , in which s is the displacement after a time t .

Worked Example 1.

Figure 1 shows the displacement-time graph for a tennis ball which is thrown vertically up in the air from a player's hand and then falls to the ground.

The graph illustrates the motion of the ball.

Taking the origin to be the starting point, the ball moves upwards (in the positive direction) to a maximum height of 2.5 m above the origin. It stops momentarily and then drops to the ground, which is 1.5 m below the initial release.

Consequently, the velocity of the ball changes from positive (on the way up) to 0 at the top and then to negative (on the way down).

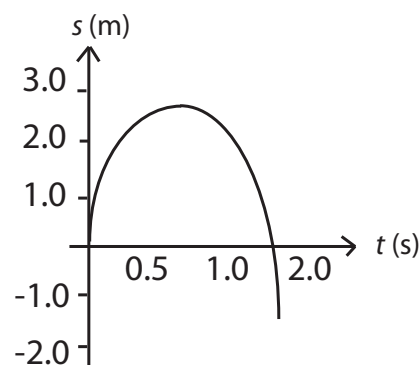


Figure 1

The gradient of a displacement-time graph is velocity

2. Velocity-time graphs.

Any point on such a graph will have coordinates (t,v) , in which v is the velocity after a time t .

Worked Example 2.

Figure 2 shows the velocity-time graph for the motion of the tennis ball described in example 1.

It was thrown into the air with a velocity of 7 m s^{-1} . It has zero velocity at 0.71 seconds and returns to the ground after 1.62 seconds with a velocity of -8.8 m s^{-1} .

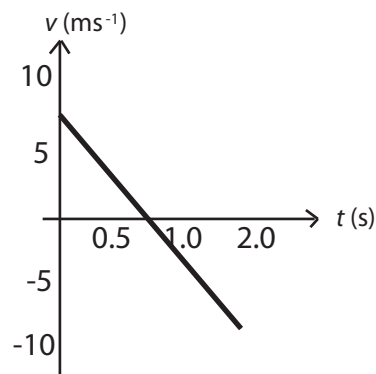


Figure 2

The gradient of a velocity-time graph is acceleration

Note that:

The 'area under a velocity-time graph' indicates the displacement

Exercises

1. A bus travels along a straight road for 600 m. It travels at a constant velocity for the whole journey, which takes 90 s. Sketch the displacement-time graph. What was the velocity of the bus?
2. A snooker ball moves in a straight line with a constant speed of $u \text{ m s}^{-1}$. It hits the cushion directly after a time t_1 and rebounds along the same path with a constant speed of $(u - 0.2) \text{ m s}^{-1}$. Sketch the displacement-time graph. (Assume $u > 0.2$)
3. Figure 3 shows the velocity-time graph for a moped, which travelled between two sets of traffic lights on a straight road.
 - (a) What is the moped's acceleration in each of the time intervals OX, XY and YZ?

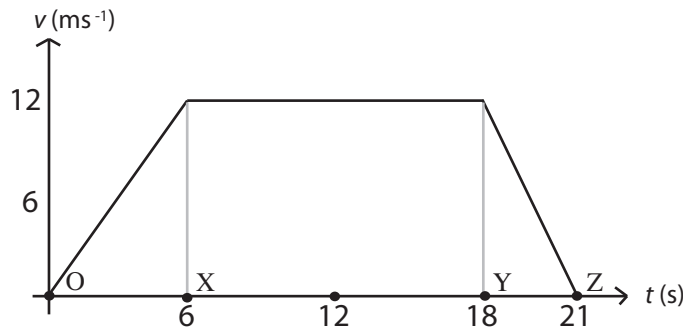
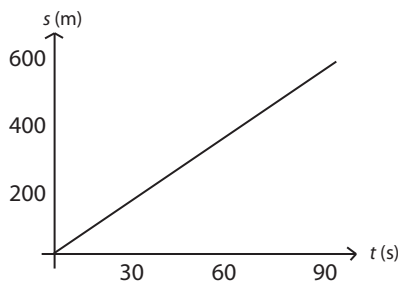


Figure 3

- (b) What was the total distance between the two sets of traffic lights?

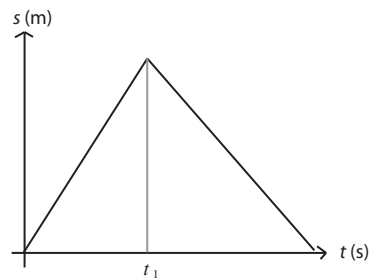
Answers (all to 2 s.f.)

1.



$$\text{Velocity} = \frac{600}{90} = 6.7 \text{ m s}^{-1}$$

2.



It is important to realise that the gradient between $t = 0$ and $t = t_1$ is steeper than that after t_1 , to reflect the greater velocity before impact with the cushion.

3. (a) OX: $a = 2.0 \text{ m s}^{-2}$, XY: $a = 0.0 \text{ m s}^{-2}$, YZ: $a = -4.0 \text{ m s}^{-2}$

3. (b) Distance = $(0.5 \times 6 \times 12) + (12 \times 12) + (0.5 \times 3 \times 12) = 198 \text{ m} = 200 \text{ m}$ (2 s.f.).