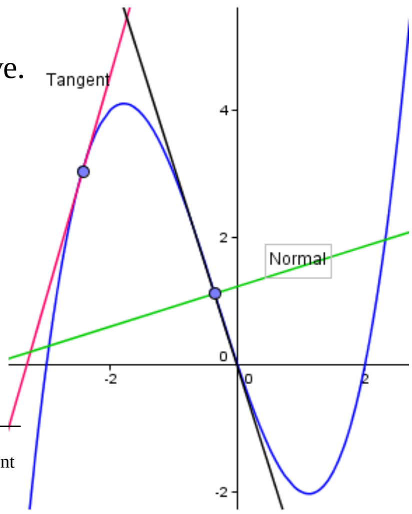
Using differentiation

Tangents and normals



- A tangent to a curve is a straight line that touches the curve. The gradient of the tangent is the same as the gradient of the curve at the point of contact.

- A normal to a curve at a point A is a straight line which is perpendicular to the tangent to the curve at A.

Consequence: $m_{\text{tangent}} \times m_{normal} = -1$ *or* $m_{normal} = \dfrac{-1}{m_{\text{tangent}}}$

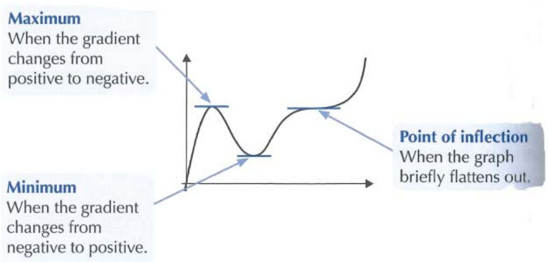
Second order derivatives

- If you differentiate y with respect to x, you get the derivative $\dfrac{dy}{dx}$.
- If you differentiate $\dfrac{dy}{dx}$ with respect to x, you get the second order derivative $\dfrac{d^2y}{dx^2}$.
- The second order derivative gives the rate of change of the gradient of the curve with respect to x.
- If $y = f(x)$, we usethe notation $\dfrac{d^2y}{dx^2} = f''(x)$.

Stationary points

Stationary points occur when the gradient of the curve is zero: $\dfrac{dy}{dx} = 0$

There are three kinds of stationary points:



To work out the coordinates of a stationary point:

1) Work out $f'(x)$
2) Solve the equation $f'(x) = 0$
3) Substitute the x-values found into the original equation to find y-values.

Minimum and maximum points

If the point $A(x_A, f(x_A))$ is a stationary point of the curve $y = f(x)$,
The nature of the point A is determined by the sign of the second order derivative:

If $\dfrac{d^2y}{dx^2}(x = x_A) < 0$, A is a maximum point

If $\dfrac{d^2y}{dx^2}(x = x_A) = 0$, A is point of inflection

If $\dfrac{d^2y}{dx^2}(x = x_A) > 0$, A is a minimum point