Collision-part 2

1D collisions

Specifications

Candidates should learn the following formulae, which are **not** included in the formulae booklet, but which may be required to answer questions.

Collision $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$$m_1\mathbf{u}_1 + m_2\mathbf{u}_2 = m_1\mathbf{v}_1 + m_2\mathbf{v}_2$$

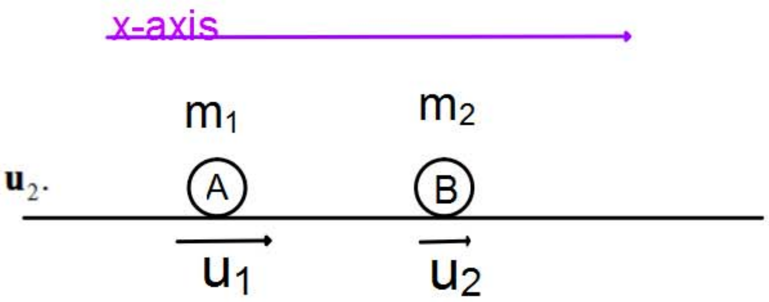
$$v = eu$$

$$v_1 - v_2 = -e(u_1 - u_2)$$

Collisions in one dimension

Conservation of momentum.	$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$
Newton's Experimental Law.	$v = eu$
Coefficient of restitution.	$v_1 - v_2 = -e(u_1 - u_2)$

Conservation of linear momentum (C.L.M)



The set-up:

Two particles A and B with mass m_1 *and* m_2 have initial speed $\mathbf{u}_1$ *and* $\mathbf{u}_2$.

The two particles are moving in the same straight direction.

These particles will/might collide.

- In the previous chapter, we have established that

$$\mathbf{F} = \frac{d\mathbf{p}}{dt}.$$

Considering both particles as the whole system,

The total momentum is $\mathbf{p} = m_1\mathbf{u}_1 + m_2\mathbf{u}_2$ (*the some of both momentums*)

and the only force applying to the system is gravity $(m_1 + m_2)\mathbf{g}$.

Since we are moving in the x-direction, $\mathbf{F} = \frac{d\mathbf{p}}{dt}$ *become*

$$\frac{dp}{dt} = 0$$ *meaning that* the total momentum is constant through time.

Momentum at t=0 — Momentum at anytime t

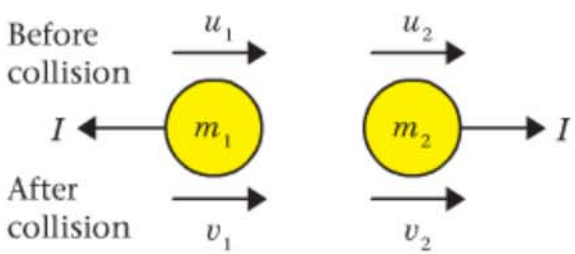
$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

- If you consider the particles individually,

each one exerts a nequal and opposite force on the other.

They are in contact for the same time,

so they exert an impulse on the other of equal magnitude but opposite in direction.



meaning :

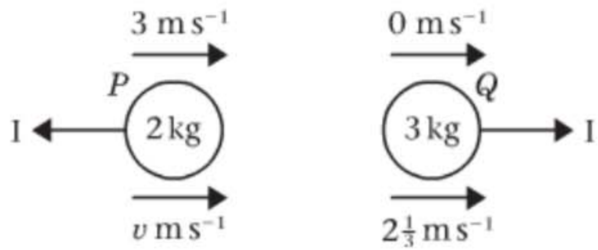
$$I_{A\to B} = (m_2v_2 - m_2u_2) = -I_{B\to A} = -(m_1v_1 - m_1u_1)$$

Example

A particle P of mass 2 kg is moving with speed $3\,\mathrm{m\,s^{-1}}$ on a smooth horizontal plane. Particle Q of mass 3 kg is at rest on the plane. Particle P collides with particle Q and after the collision Q moves off with speed $2\frac{1}{3}\,\mathrm{m\,s^{-1}}$. Find

a the speed and direction of motion of P after the collision,

b the magnitude of the impulse received by P in the collision.



Exercises:

1 A particle P of mass 2 kg is moving on a smooth horizontal plane with speed $4\,\mathrm{m\,s^{-1}}$. It collides with a second particle Q of mass 1 kg which is at rest. After the collision P has speed $2\,\mathrm{m\,s^{-1}}$ and it continues to move in the same direction. Find the speed of Q after the collision.

2 A railway truck of mass 25 tonnes moving at $4\,\mathrm{m\,s^{-1}}$ collides with a stationary truck of mass 20 tonnes. As a result of the collision the trucks couple together. Find the common speed of the trucks after the collision.

3 Particles A and B have mass 0.5 kg and 0.2 kg respectively. They are moving with speeds $5\,\mathrm{m\,s^{-1}}$ and $2\,\mathrm{m\,s^{-1}}$ respectively in the same direction along the same straight line on a smooth horizontal surface when they collide. After the collision A continues to move in the same direction with speed $4\,\mathrm{m\,s^{-1}}$. Find the speed of B after the collision.

3 $4.5\,\mathrm{m\,s^{-1}}$
2 $2\frac{2}{9}\,\mathrm{m\,s^{-1}}$
1 $4\,\mathrm{m\,s^{-1}}$

Newton's law of restitution

Newton's law of impact If two objects moving in the same straight line collide, the ratio $\frac{\text{separation speed}}{\text{approach speed}}$ is constant.

This value of the constant, called the **coefficient of restitution**, depends on the shape of the objects and the materials they consist of.

The value of the coefficient of restitution e depends on the materials from which the particles are made. Particles for which $e = 1$ are called **perfectly elastic** particles. Particles for which $e = 0$ are called **inelastic particles**. Inelastic particles coalesce on impact.

Although Newton's law of impact has been discussed in terms of the collision of two spheres, it is often applied to collisions between objects of other shapes.

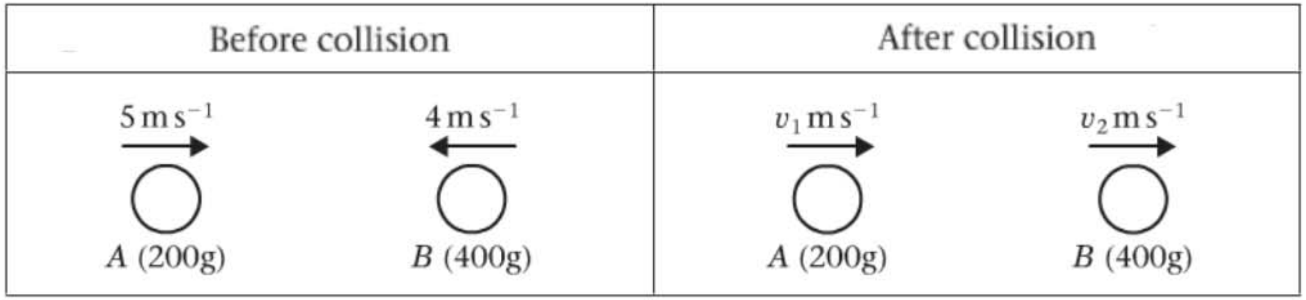
Some people like to remember the law in the algebraic form

$$v_2 - v_1 = -e(u_2 - u_1).$$

You can solve problems involving the direct impact of two particles by using conservation of linear momentum and Newton's Law of Restitution.

Example

Find the values of v_1 and v_2 in the situation shown, given that the coefficient of restitution e is $\frac{1}{2}$.



$v_1 = -4$
$v_2 = \frac{1}{2}$

Exercises:

2 In each part of this question the two diagrams show the speeds of two particles A and B just before and just after a collision. The particles move on a smooth horizontal plane. The masses of A and B and the coefficients of restitution e are also given. Find the values of v_1 and v_2 in each case.

	Before collision		After collision	
a $e = \frac{1}{2}$	$6\,\text{m s}^{-1}$ → A (0.25 kg)	At rest B (0.5 kg)	$v_1\,\text{m s}^{-1}$ → A (0.25 kg)	$v_2\,\text{m s}^{-1}$ → B (0.5 kg)
b $e = 0.25$	$4\,\text{m s}^{-1}$ → A (2 kg)	$2\,\text{m s}^{-1}$ → B (3 kg)	$v_1\,\text{m s}^{-1}$ → A (2 kg)	$v_2\,\text{m s}^{-1}$ → B (3 kg)
c $e = \frac{1}{7}$	$8\,\text{m s}^{-1}$ → A (3 kg)	$6\,\text{m s}^{-1}$ ← B (1 kg)	$v_1\,\text{m s}^{-1}$ → A (3 kg)	$v_2\,\text{m s}^{-1}$ → B (1 kg)
d $e = \frac{2}{3}$	$6\,\text{m s}^{-1}$ → A (400 g)	$6\,\text{m s}^{-1}$ ← B (400 g)	$v_1\,\text{m s}^{-1}$ → A (400 g)	$v_2\,\text{m s}^{-1}$ → B (400 g)

3 A small smooth sphere A of mass 1 kg is travelling along a straight line on a smooth horizontal plane with speed $4\,\text{m s}^{-1}$ when it collides with a second smooth sphere B of the same radius, with mass 2 kg and travelling in the same direction as A with speed $2.5\,\text{m s}^{-1}$. After the collision, A continues in the same direction with speed $2\,\text{m s}^{-1}$.

a Find the speed of B after the collision.

b Find the coefficient of restitution for the spheres.

4 Two spheres A and B are of equal radius and have masses 2 kg and 6 kg respectively. A and B move towards each other along the same straight line on a smooth horizontal surface with velocities $4\,\text{m s}^{-1}$ and $6\,\text{m s}^{-1}$ respectively. If the coefficient of friction is $\frac{1}{5}$, find the velocities of the spheres after the collision and the magnitude of the impulse given to each sphere.

5 Two particles of masses $2m$ and $3m$ respectively are moving towards each other with speed u. If the $3m$ mass is brought to rest by the collision, find the speed of the $2m$ mass after the collision and the coefficient of restitution between the particles.

6 Two particles A and B are travelling along the same straight line in the same direction on a smooth horizontal surface with speeds $3u$ and u respectively. Particle A catches up and collides with particle B. If the mass of B is twice that of A and the coefficient of restitution is e find, in terms of e and u, expressions for the speeds of A and B after the collision.

2 **a** $v_1 = 0$, $v_2 = 3$ **b** $v_1 = 2\frac{1}{2}$, $v_2 = 3$
c $v_1 = 4$, $v_2 = 6$ **d** $v_1 = -4$, $v_2 = 4$
e $v_1 = -5$, $v_2 = -2$

3 **a** $3.5\,\text{m s}^{-1}$ **b** 1

4 $5\,\text{m s}^{-1}$ and $3\,\text{m s}^{-1}$ both in the direction that B was moving before the impact.
18 N s

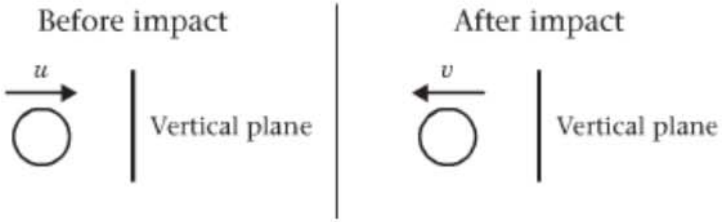
5 $\frac{u}{2}\,\text{m s}^{-1}$, $\frac{1}{4}$

6 $\frac{u}{3}(5 - 4e)$, $\frac{u}{3}(5 + 2e)$

Collision "with a wall"

You can also apply Newton's Law of Restitution to problems involving direct collision of a particle with a smooth plane surface perpendicular to the direction of motion of the particle.

In the figure a particle is shown moving horizontally with speed u before impact with a vertical plane surface. After impact the particle moves in the opposite direction with speed v.



- $e = \dfrac{\text{speed of rebound}}{\text{speed of approach}}$
- $e = \dfrac{v}{u}$

Example

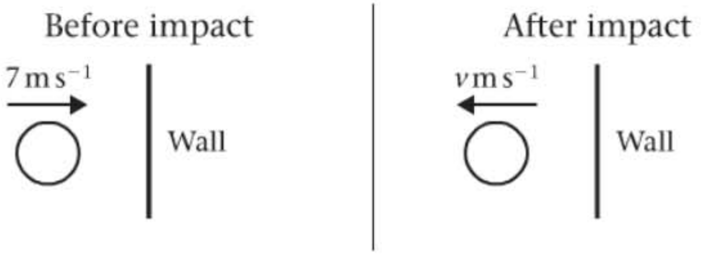
A small sphere collides normally with a fixed vertical wall. Before the impact the sphere is moving with a speed of $4\,\text{m}\,\text{s}^{-1}$ on a smooth horizontal floor. The coefficient of restitution between the sphere and the wall is 0.2. Find the speed of the sphere after the collision.

The speed of the sphere after the collision is $0.8\,\text{m}\,\text{s}^{-1}$

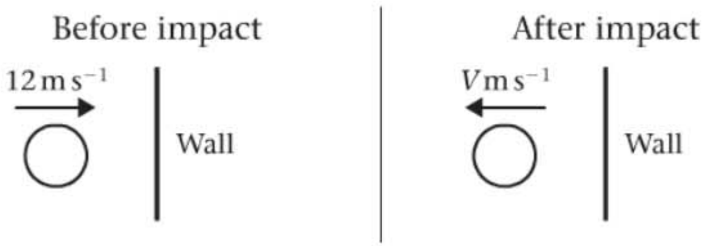
Exercises:

A smooth sphere collides normally with a fixed vertical wall. The two diagrams show the speed of the sphere before and after the collision. The value of e is given in each case.
Find the speed of the sphere after the collision in each case.

a $e = \frac{1}{2}$



b $e = \frac{1}{4}$



v=3.5m/s
V=3m/s

Mixed questions

3 Three perfectly elastic particles A, B and C of masses $3m$, $5m$ and $4m$ respectively lie at rest on a straight line on a smooth horizontal table with B between A and C. Particle A is projected directly towards B with speed $6\,\text{m s}^{-1}$ and after A has collided with B, B then collides with C. Find the speed of each particle after the second impact.

4 Three identical smooth spheres A, B and C, each of mass m, lie at rest on a straight line on a smooth horizontal table. Sphere A is projected with speed u to strike sphere B directly. Sphere B then strikes sphere C directly. The coefficient of restitution between any two spheres is e, $e \neq 1$.

a Find the speeds in terms of u and e of the spheres after these two collisions.

b Show that A will catch up with B and there will be a further collision.

5 Three identical spheres A, B and C of equal mass m, and equal radius move along the same straight line on a horizontal plane. B is between A and C. A and B are moving towards each other with velocities $4u$ and $2u$ respectively while C moves away from B with velocity $3u$.

a If the coefficient of restitution between any two of the spheres is e, show that B will only collide with C if $e > \frac{2}{3}$.

b Find the direction of motion of A after collision, if $e > \frac{2}{3}$.

6 Two particles P of mass $2m$ and Q of mass $3m$ are moving towards each other with speeds $4u$ and $2u$ respectively. The direction of motion of Q is reversed by the impact and its speed after impact is u. This particle then hits a smooth vertical wall perpendicular to its direction of motion. The coefficient of restitution between Q and the wall is $\frac{2}{3}$. In the subsequent motion, there is a further collision between Q and P. Find the speeds of P and Q after this collision.

AQA-exam question

Two particles, A and B, are moving towards each other along a straight, horizontal line. Particle A has mass 13 kg and speed $5\ \text{m s}^{-1}$. Particle B has mass 7 kg and speed $3\ \text{m s}^{-1}$. The coefficient of restitution between the two particles is 0.4. The two particles collide.

(a) Show that the speed of B after the collision is $4.28\ \text{m s}^{-1}$. *(6 marks)*

(b) Find the speed of A after the collision. *(2 marks)*

(c) State, giving a reason for your answer, which of the two particles changes direction as a result of the collision. *(1 mark)*

(d) Calculate the magnitude of the impulse on B during the collision. *(2 marks)*

3 $-1.5\,\text{m s}^{-1}$, $0.5\,\text{m s}^{-1}$ and $5\,\text{m s}^{-1}$

4 a $\frac{1}{2}u(1-e)$, $\frac{1}{4}u(1+e)(1-e)$ and $\frac{1}{4}u(1+e)^2$

b A will catch up with B provided that $2 > 1 + e$. Since $e < 1$ this condition holds and A will catch up with B, resulting in a further collision.

5 a $u(1+3e) > 3u \Rightarrow e > \frac{2}{3}$

b A moves away from B.

6 $\frac{5u}{8}$ and $\frac{7u}{12}$

(a)	$13\times5+7\times(-3)=13v_A+7v_B$		
	$44=13v_A+7v_B$	M1A1	
	$v_A-v_B=-0.4(5-(-3))$		
	$v_A-v_B=-3.2$	M1A1	
	$v_A=v_B-3.2$		
	$44=13(v_B-3.2)+7v_B$	M1	
	$v_B=\frac{85.6}{20}=4.28$	A1	6
(b)	$v_A=4.28-3.2=1.08$	M1A1	2
(c)	B, as the sign of the velocity changes during the collision.	B1	1
(d)	$I=7\times4.28-7\times(-3)=50.96\ \text{N s}$	M1A1	2
	Total		11