1D Collisions – exam questions

Question 1: June 2006 – Q2

Three smooth spheres A, B and C of equal radii and masses m, m and $2m$ respectively lie at rest on a smooth horizontal table. The centres of the spheres lie in a straight line with B between A and C. The coefficient of restitution between any two spheres is e.

The sphere A is projected directly towards B with speed u and collides with B.

(a) Find, in terms of u and e, the speed of B immediately after the impact between A and B. *(5 marks)*

(b) The sphere B subsequently collides with C. The speed of C immediately after this collision is $\frac{3}{8}u$. Find the value of e. *(7 marks)*

Question 2: June 2007 – Q4

Two small smooth spheres, A and B, of equal radii have masses 0.3 kg and 0.2 kg respectively. They are moving on a smooth horizontal surface directly towards each other with speeds $3\,\text{m s}^{-1}$ and $2\,\text{m s}^{-1}$ respectively when they collide. The coefficient of restitution between A and B is 0.8.

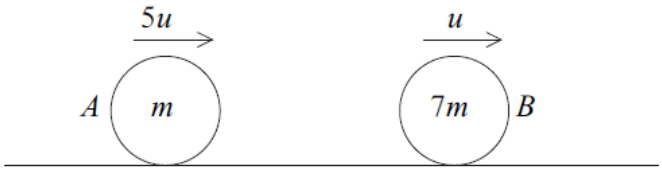
(a) Find the speeds of A and B immediately after the collision. *(6 marks)*

(b) Subsequently, B collides with a fixed smooth vertical wall which is at right angles to the path of the sphere. The coefficient of restitution between B and the wall is 0.7.

Show that B will collide again with A. *(3 marks)*

Question 3: June 2009 – Q6

A smooth sphere A of mass m is moving with speed $5u$ in a straight line on a smooth horizontal table. The sphere A collides directly with a smooth sphere B of mass $7m$, having the same radius as A and moving with speed u in the same direction as A. The coefficient of restitution between A and B is e.



Before collision

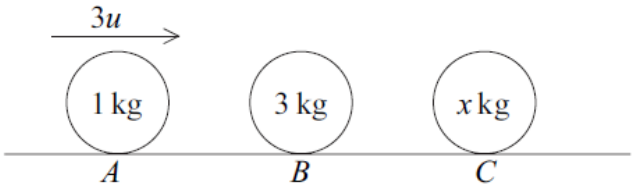
(a) Show that the speed of B after the collision is $\frac{u}{2}(e+3)$. *(5 marks)*

(b) Given that the direction of motion of A is reversed by the collision, show that $e > \frac{3}{7}$. *(4 marks)*

(c) Subsequently, B hits a wall fixed at right angles to the direction of motion of A and B. The coefficient of restitution between B and the wall is $\frac{1}{2}$. Given that after B rebounds from the wall both spheres move in the same direction and collide again, show also that $e < \frac{9}{13}$. *(4 marks)*

Question 4: June 2010 – Q3

Three smooth spheres, A, B and C, of equal radii have masses 1 kg, 3 kg and x kg respectively. The spheres lie at rest in a straight line on a smooth horizontal surface with B between A and C. The sphere A is projected with speed $3u$ directly towards B and collides with it.



The coefficient of restitution between each pair of spheres is $\frac{1}{3}$.

(a) Show that A is brought to rest by the impact and find the speed of B immediately after the collision in terms of u. *(6 marks)*

(b) Subsequently, B collides with C.

Show that the speed of C immediately after the collision is $\frac{4u}{3+x}$.

Find the speed of B immediately after the collision in terms of u and x. *(6 marks)*

(c) Show that B will collide with A again if $x > 9$. *(2 marks)*

(d) Given that $x = 5$, find the magnitude of the impulse exerted on C by B in terms of u. *(2 marks)*

Question 5: June 2011 – Q5

A ball is dropped from a height of 2.5 m above a horizontal floor. The ball bounces repeatedly on the floor.

(a) Find the speed of the ball when it first hits the floor. *(2 marks)*

(b) The coefficient of restitution between the ball and the floor is e.

(i) Show that the time taken between the first contact of the ball with the floor and the second contact of the ball with the floor is $\frac{10e}{7}$ seconds. *(3 marks)*

(ii) Find, in terms of e, the time taken between the second contact and the third contact of the ball with the floor. *(1 mark)*

(c) Find, in terms of e, the total vertical distance travelled by the ball from when it is dropped until its third contact with the floor. *(5 marks)*

(d) State a modelling assumption for answering this question, other than the ball being a particle. *(1 mark)*

1D Collisions – exam questions - MS

Question 1: June 2006 – Q2

(a)	conservation of momentum		
	$mu = mv_A + mv_B$	M1	
	$u = v_A + v_B$	A1	
	restitution		
	$eu = v_B - v_A$	M1A1	
	$v_B = \frac{1}{2}u(1+e)$	A1F	5
(b)	$mv_B = mw_B + 2m\frac{3u}{8}$	M1A1	
	$ev_B = \frac{3u}{8} - w_B$	M1A1	
	Elimination of w_B	m1	
	$4e^2 + 8e - 5 = 0$	A1F	
	$e = \frac{1}{2}$	A1F	7
	Total		**12**

Question 2: June 2007 – Q4

(a)	Conservation of momentum :		
	$0.3(3) - 0.2(2) = 0.3v_A + 0.2v_B$	M1A1	
	$3v_A + 2v_B = 5$ ----------(1)		
	Newton's experimental law :		
	$0.8 = \frac{v_B - v_A}{5}$	M1	
	$v_B - v_A = 4$ ----------(2)	A1	
	Solving (1) and (2)	m1	
	$v_B = 3.4$		
	$v_A = -0.6$	A1F	6
(b)	$0.7 = \frac{v}{3.4}$	M1	
	$v = 2.38$	A1F	
	Speed of B (2.38) $\succ$ Speed of A (0.6)		
	$\therefore$ B collides again with A	E1	3
	Total		**9**

Question 3: June 2009 – Q6

(a)	$5mu + 7mu = mv_A + 7mv_B$	M1A1	
	$12u = v_A + 7v_B$		
	$e = \frac{-v_A + v_B}{4u}$	M1	
	$-v_A + v_B = 4eu$		
	$8v_B = 12u + 4eu$	m1	
	$v_B = \frac{u}{2}(e+3)$	A1	5
(b)	$v_A = \frac{u}{2}(e+3) - 4eu$	M1	
	$v_A = \frac{u}{2}(3-7e)$	A1F	
	$\frac{u}{2}(3-7e) < 0$	M1	
	$3 - 7e < 0$		
	$e > \frac{3}{7}$	A1	4
(c)	$w_B = \frac{u}{4}(e+3)$	M1	
	$\frac{u}{2}(7e-3) < \frac{u}{4}(e+3)$	M1	
	$2(7e-3) < e+3$		
	$13e < 9$	m1	
	$e < \frac{9}{13}$	A1	4
	Total		**13**

Question 4: June 2010 – Q3

(a)	C.L.M.		
	$(1)3u = (1)v_A + (3)v_B$	M1 A1	
	Restitution :		
	$\frac{1}{3} \times 3u = v_B - v_A$	M1 A1	
	$v_B = u$	m1	
	$v_A = 0$	A1	6
(b)	C.L.M.		
	$3u = 3w_B + xw_C$	M1 A1	
	Restitution :		
	$\frac{1}{3}u = w_C - w_B$	M1 A1	
	$w_C = \frac{4u}{3+x}$	m1	
	$w_B = \frac{u(9-x)}{3(3+x)}$ OE	A1	6
(c)	For further collision $\frac{u(9-x)}{3(3+x)} < 0$	M1	
	$9u - xu < 0$		
	$x > 9$	A1	2
(d)	$I = 5(\frac{4u}{3+5})$	M1	
	$I = \frac{5u}{2}$	A1	2
	Alternative:		
	$I = 3u - 3 \times \frac{u(9-5)}{3(3+5)}$	(M1)	
	$I = \frac{5u}{2}$	(A1F)	
	Total		**16**

Question 5: June 2011 – Q5

5(a)	$v^2 = u^2 + 2as$		
	$v^2 = 0^2 + 2(9.8)(2.5)$	M1	
	$v = 7$	A1	2
(b)(i)	$\frac{w}{7} = e$	M1	
	$w = 7e$		
	$0 = 7et - \frac{9.8}{2}t^2$ **or** $(0 = 7e - 9.8t)$	M1	
	$t = \frac{10e}{7}$ $(t = 2 \times \frac{7e}{9.8})$	A1	3
(ii)	$w' = 7e^2$		
	$0 = 7e^2 t' - \frac{9.8}{2}t'^2$		
	$t' = \frac{10e^2}{7}$	B1	1
(c)	$0^2 = (7e)^2 + 2(-9.8)h_2$	M1	
	$h_2 = 2.5e^2$	A1	
	$h_3 = 2.5e^2$		
	$0^2 = (7e^2)^2 + 2(-9.8)h_4$		
	$h_4 = 2.5e^4$	A1	
	$h_5 = 2.5e^4$		
	Total distance $= 2.5 + 2(2.5e^2) + 2(2.5e^4)$	m1	
	$= 2.5 + 5e^2 + 5e^4$	A1	5