

Circles

Equation of a circle.

Consider a circle C with centre $\Omega(a,b)$ and radius r .

Any point $M(x,y)$ on this circle satisfies $\Omega M = r$.

This is equivalent to $\Omega M^2 = r^2$

$$(x-a)^2 + (y-b)^2 = r^2$$

In particular if the centre is $(0,0)$, the equation becomes $x^2 + y^2 = r^2$.



Re-arranging circle equations

In its factorised form, it is easy to read centre and radius from the equation of a circle:

$(x-3)^2 + (y+1)^2 = 9$ is the equation of the circle centre $(3,-1)$ radius 3.

But an equation can be given in its expanded form: $x^2 + y^2 + 2ax + 2by + c = 0$.

To re-arrange this equation, use the **complete square form** with $x^2 + 2ax$ and $y^2 + 2by$.

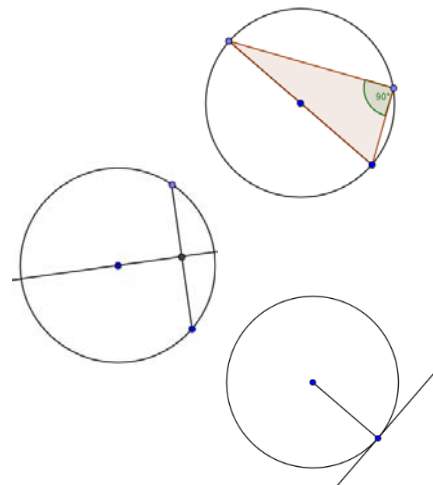
example :

$$\begin{aligned}x^2 + y^2 + 6x + 4y - 3 &= 0 \\x^2 + 6x + y^2 + 4y - 3 &= 0 \\(x+3)^2 - 9 + (y+2)^2 - 4 - 3 &= 0 \\(x+3)^2 + (y+2)^2 &= 16\end{aligned}$$

This is the circle centre $(-3,-2)$ radius $r = 4$

Circle properties

- Any point joined to the extremities of a diameter form a right-angled triangle.
- The perpendicular bisector of a chord goes through the centre of the circle.
- A tangent to the circle is perpendicular to the radius at its point of contact.



Work out exercise: Tangent to a circle

The circle C has centre $O(2,1)$ and radius 25.

The point $A(6,4)$ belongs to the circle.

Work out the equation of the tangent to the circle at A.

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- The tangent at A is PERPENDICULAR to the radius OA.

The gradient of OA is $m_1 = \frac{4-1}{2-6} = \frac{3}{-4}$ The gradient of the tangent is therefore $-\frac{1}{m_1} = \frac{4}{3}$

The equation of the tangent is : $y - 4 = \frac{4}{3}(x - 6)$

$$3y - 12 = 4x - 24$$

$$4x - 3y - 12 = 0$$

