Circles

Equation of a circle.

Consider a circle C with centre $\Omega(a,b)$ and radius r.

Any point $M(x,y)$ on this circle satisfies $\Omega M = r$.

This is equivalent to $\Omega M^2 = r^2$

$$(x-a)^2+(y-b)^2=r^2$$

In particular if the centre is (0,0), the equation becomes $x^2+y^2=r^2$.

Re-arranging circle equations

In its factorised form, it is easy to read centre and radius from the equation of a circle:

$(x-3)^2+(y+1)^2=9$ is the eqation of the circle centre (3,-1) radius 3.

But an equation can be given in its expanded form: $x^2+y^2+2ax+2by+c=0$.

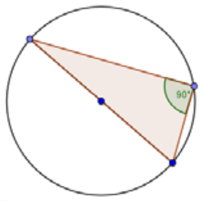
To re-arrange this equation, use the complete square form with x^2+2ax *and* y^2+2bx.

example:

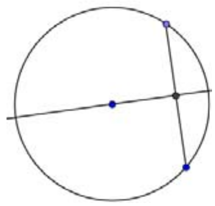
$$x^2+y^2+6x+4y-3=0$$
$$x^2+6x \quad + \quad y^2+4y \quad -3=0$$
$$(x+3)^2-9 \quad + \quad (y+2)^2-4 \quad -3=0$$
$$(x+3)^2+(y+2)^2=16$$

This is the circle centre $(-3,-2)$ radius $r=4$

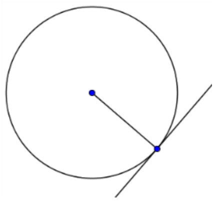
Circle properties



a) Any point joined to the extremities of a diameter form a right-angled triangle.



b) The perpendicular bisector of a chord goes through the centre of the circle.



c) A tangent to the circle is perpendicular to the radius at its point of contact.

Work out exercise: Tangent to a circle

The circle C has centre O(2,1) and radius 25.

The point A(6,4) belongs to the circle.

Work out the equation of the tangent to the circle at A.

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•The tangent at A is PERPENDICULAR to the radius OA.

The gradient of OA is $m_1 = \frac{4-1}{2-6} = \frac{3}{-4}$ The gradient of the tangent is therefore $-\frac{1}{m_1} = \frac{4}{3}$

The equation of the tangent is : $y-4=\frac{4}{3}(x-6)$

$$3y-12=4x-24$$
$$4x-3y-12=0$$