Normal distribution - Exam questions

Question 1: Jan 2006

(a) The weight, X grams, of soup in a carton may be modelled by a normal random variable with mean 406 and standard deviation 4.2.

Find the probability that the weight of soup in a carton:

(i) is less than 400 grams; *(3 marks)*

(ii) is between 402.5 grams and 407.5 grams. *(4 marks)*

(b) The weight, Y grams, of chopped tomatoes in a tin is a normal random variable with mean μ and standard deviation σ.

(i) Given that $P(Y < 310) = 0.975$, explain why:

$$310 - \mu = 1.96\sigma$$ *(3 marks)*

(ii) Given that $P(Y < 307.5) = 0.86$, find, to two decimal places, values for μ and σ. *(4 marks)*

Question 2: Jan 2008

In large-scale tree-felling operations, a machine cuts down trees, strips off the branches and then cuts the trunks into logs of length X metres for transporting to a sawmill.

It may be assumed that values of X are normally distributed with mean μ and standard deviation 0.16, where μ can be set to a specific value.

(a) Given that μ is set to 3.3, determine:

(i) $P(X < 3.5)$; *(3 marks)*

(ii) $P(X > 3.0)$; *(3 marks)*

(iii) $P(3.0 < X < 3.5)$. *(2 marks)*

(b) The sawmill now requires a batch of logs such that there is a probability of 0.025 that any given log will have a length less than 3.1 metres.

Determine, to two decimal places, the new value of μ. *(4 marks)*

Question 3: Jun 2008

When a particular make of tennis ball is dropped from a vertical distance of 250 cm on to concrete, the height, X centimetres, to which it first bounces may be assumed to be normally distributed with a mean of 140 and a standard deviation of 2.5.

(a) Determine:

(i) $P(X < 145)$; *(3 marks)*

(ii) $P(138 < X < 142)$. *(4 marks)*

(b) Determine, to one decimal place, the maximum height exceeded by 85% of first bounces. *(4 marks)*

~~(c) Determine the probability that, for a random sample of 4 first bounces, the mean height is greater than 139 cm.~~ *(4 marks)*

Question 4: Jan 2007

When Monica walks to work from home, she uses either route A or route B.

(a) Her journey time, X minutes, by route A may be assumed to be normally distributed with a mean of 37 and a standard deviation of 8.

Determine:

(i) $\mathrm{P}(X < 45)$; *(3 marks)*

(ii) $\mathrm{P}(30 < X < 45)$. *(3 marks)*

(b) Her journey time, Y minutes, by route B may be assumed to be normally distributed with a mean of 40 and a standard deviation of σ.

Given that $\mathrm{P}(Y > 45) = 0.12$, calculate the value of σ. *(4 marks)*

(c) If Monica leaves home at 8.15 am to walk to work hoping to arrive by 9.00 am, state, with a reason, which route she should take. *(2 marks)*

Question 5: Jun 2010

Each day, Margot completes the crossword in her local morning newspaper. Her completion times, X minutes, can be modelled by a normal random variable with a mean of 65 and a standard deviation of 20.

(a) Determine:

(i) $\mathrm{P}(X < 90)$;

(ii) $\mathrm{P}(X > 60)$. *(5 marks)*

Question 6: Jun 2009

The weight, X grams, of talcum powder in a tin may be modelled by a normal distribution with mean 253 and standard deviation σ.

(a) Given that $\sigma = 5$, determine:

(i) $\mathrm{P}(X < 250)$; *(3 marks)*

(ii) $\mathrm{P}(245 < X < 250)$; *(2 marks)*

(iii) $\mathrm{P}(X = 245)$. *(1 mark)*

(b) Assuming that the value of the mean remains unchanged, determine the value of σ necessary to ensure that 98% of tins contain more than 245 grams of talcum powder. *(4 marks)*

Normal distribution - Exam questions - MS

Question 1: Jan 2006

Weight, $X \sim N(406, 4.2^2)$

$$P(X < 400) = P\left(Z < \frac{400-406}{4.2}\right)$$

$= P(Z < -1.428 \text{ to } -1.43)$

$= 1 - P(Z < 1.428 \text{ to } 1.43)$

$= \boxed{0.076 \text{ to } 0.077}$

$P(402.5 < X < 407.5) =$

$P(X < 407.5) - P(X < 402.5) =$

$P(Z < 0.36) - P(Z < -0.83)$

$= 0.64058 - (1 - 0.79673) = \boxed{0.433 \text{ to } 0.44}$

$0.975 \quad \Rightarrow \quad \boxed{z = 1.96}$

$$P(Y < 310) = P\left(Z < \frac{310-\mu}{\sigma}\right)$$

or

$x = \mu \; +/\pm \; z\sigma$

Thus $\boxed{\frac{310-\mu}{\sigma} = 1.96} \quad \Rightarrow \quad$ result

or

$310 = \mu + 1.96\sigma \quad \Rightarrow \quad$ result

$0.86 \quad \Rightarrow \quad \boxed{z = 1.08}$

$310 \quad - \; \mu = 1.96\sigma$

$307.5 - \mu = 1.08\sigma$

$2.5 = 0.88\sigma$

$\boxed{\sigma = 2.84 \text{ to } 2.842}$

$\boxed{\mu = 304.4 \text{ to } 304.5}$

Question 2: Jan 2008

$$P(X < 3.5) = P\left(Z < \frac{3.5-3.3}{0.16}\right) =$$

$P(Z < 1.25) =$

$\boxed{0.894 \text{ to } 0.895}$

$$P(X > 3.0) = P\left(Z > \frac{3.0-3.3}{0.16}\right) =$$

$P(Z > -1.875) = P(Z < 1.875) =$

$\boxed{0.969 \text{ to } 0.97(0)}$

$P(3.0 < X < 3.5) = \text{(i)} - [1 - \text{(ii)}] =$

$\boxed{0.863 \text{ to } 0.865}$

$0.025 \Rightarrow z = 1.96$

$$z = \frac{3.1-\mu}{0.16}$$

$= -1.96$

Hence $\boxed{\mu = 3.4(0) \text{ to } 3.42}$

Question 3: Jun 2008

$$P(X < 145) = P\left(Z < \frac{145-140}{2.5}\right) =$$

$P(Z < 2) =$

$\boxed{0.977 \text{ to } 0.98(0)}$

$P(138 < X < 142) =$

$P(X < 142) - P(X < 138) =$

$P(Z < 0.8) - P(Z < -0.8) =$

$P(Z < 0.8) - \{1 - P(Z < 0.8)\} =$

$(0.78814) - (1 - 0.78814) =$

$\boxed{0.576 \text{ to } 0.58(0)}$

$0.85\ (85\%) \Rightarrow z = -1.03 \text{ to } -1.04$

$$z = \frac{x-140}{2.5}$$

$= \pm 1.03 \text{ to } \pm 1.04$

Hence $\boxed{x = 137.3 \text{ to } 137.5}$

Question 4: Jan 2007

$0.12 \Rightarrow z = 1.17$ to 1.18

$\text{P}(X < 45) = \text{P}\left(Z < \frac{45-37}{8}\right)$

$= \text{P}(Z < 1)$

$\boxed{= 0.841}$

$\text{P}(30 < X < 45) = \text{(i)} - \text{P}(X < 30)$

$= \text{(i)} - \text{P}(Z < -0.875)$

$= \text{(i)} - [1 - (0.80785 \text{ to } 0.81057)]$

$\boxed{= 0.648 \text{ to } 0.652}$

$z = \frac{45-40}{\sigma}$

$= 1.175$

$\boxed{\sigma = 4.23 \text{ to } 4.28}$

Route A: $\text{P}(X > 45) = 1 - \text{(a)(i)}$
Route B: $\text{P}(Y > 45) = 0.12$
so
Monica should use **Route B** (smaller prob)

Question 5: Jun 2010

Time, $X \sim \text{N}(65,\ 20^2)$

$\text{P}(X < 90) = \text{P}\left(Z < \frac{90-65}{20}\right) - \left[\text{P}\left(Z < \frac{0-65}{20}\right) = \text{P}(Z < -3.25) = 0.00058\right]$

$= \text{P}(Z < \mathbf{1.25})$

$\boxed{= \mathbf{0.893 \text{ to } 0.895}}$

$\text{P}(X > 60) = \text{P}(Z > -0.25)$

$= \text{P}(Z < 0.25)$

$= \boxed{\mathbf{0.598 \text{ to } 0.599}}$

Question 6: Jun 2009

$X \sim \text{N}(253,\ 5^2)$

$\text{P}(X < 250) = \text{P}\left(Z < \frac{250-253}{5}\right) =$

$\text{P}(Z < -0.6) = 1 - \text{P}(Z < 0.6)$

$= 1 - 0.72575$

$\boxed{= 0.274 \text{ to } 0.275}$

$\text{P}(245 < X < 250) = [\text{C's(a)(i)}] - \text{P}(X < 245)$

$= \text{(i)} - \text{P}(Z < -1.6) = 0.27425 - 0.0548$

$\boxed{= 0.219 \text{ to } 0.22(0)}$

$\boxed{\text{P}(X = 245) = 0}$ or zero or impossible

98% (0.98) $\Rightarrow z = -2.05$ to -2.06

$z = \frac{245-253}{\sigma}$

$= -2.0537$

$\boxed{\sigma = 3.88 \text{ to } 3.9(0)}$