# Vectors

*Magnitude and direction of a vector. Resultant of vectors may also be required.*

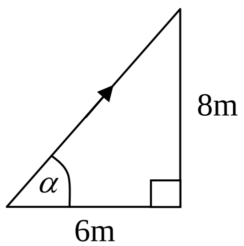
A vector is a quantity that has both direction and magnitude. So velocity is a vector quantity.

We use the vector **i** to represent the vector of magnitude 1 in the positive *x*-direction and **j** to represent the vector of magnitude 1 in the positive *y*-direction.

We can represent any vector using **i** and **j**.

Suppose a boat moved with velocity $6\mathbf{i}+8\mathbf{j}\text{ ms}^{-1}$ where the unit vectors **i** and **j** are directed due east and north respectively.

This means that each second the boat moves 6m to the East and 8m to the North each second.




By Pythagoras we can see that the boat travels at a speed of $\sqrt{6^2+8^2}=10\text{ ms}^{-1}$ .

We could also calculate the angle $\alpha$ to be $\tan^{-1}\left(\frac{8}{6}\right)=53.1^\circ$ and so the bearing is $037^\circ$.

Distance is the magnitude of displacement. If a particle has a position vector $p\mathbf{i}+q\mathbf{j}$ m relative to O then its distance from O is $\sqrt{p^2+q^2}$ m.

In a similar way, speed is the magnitude of velocity. If a particle has velocity $p\mathbf{i}+q\mathbf{j}\text{ ms}^{-1}$ then it has speed $\sqrt{p^2+q^2}\text{ ms}^{-1}$.

Suppose we were told that a particle had speed $26\text{ ms}^{-1}$ and was travelling in a direction parallel to the vector $5\mathbf{i}+12\mathbf{j}$.

Since it is travelling parallel to the vector $5\mathbf{i}+12\mathbf{j}$ the velocity must be a multiple of $5\mathbf{i}+12\mathbf{j}$.
$5\mathbf{i}+12\mathbf{j}$ has magnitude $\sqrt{5^2+12^2}=13$. We are told the particle has speed $26\text{ ms}^{-1}$ so its velocity.
Since $26=2\times13$ we see that the velocity must be $2\times(5\mathbf{i}+12\mathbf{j})=10\mathbf{i}+24\mathbf{j}\text{ ms}^{-1}$.

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*Application of vectors to displacements, velocities, accelerations and forces in a plane. Candidates may be required to resolve a vector into two components or use a vector diagram. Questions may be set involving the unit vectors* **i** *and* **j***. Use of* $\text{velocity} = \dfrac{\text{change of displacement}}{\text{time}}$ *in the case of constant velocity, and of* $\text{acceleration} = \dfrac{\text{change of velocity}}{\text{time}}$ *in the case of time constant acceleration, will be required.*

E.g. A particle *P* moves in a straight line with constant velocity. Initially *P* is at the point *A* with position vector $4\mathbf{i} + \mathbf{j}$ m relative to a fixed origin *O*, and 3 s later it is at the point *B* with position vector $-5\mathbf{i} + 13\mathbf{j}$ m.

(a) Find the velocity of *P*.

(b) Find, in degrees to one decimal place, the size of the angle between the direction of motion of *P* and the vector **i**.

Two seconds after it passes *B* the particle *P* reaches the point *C*.

(c) Find, in m to one decimal place, the distance *OC*.

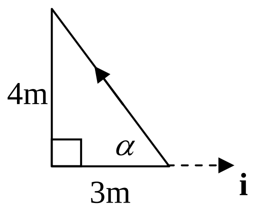
(a)

$$\text{velocity} = \frac{\text{change of displacement}}{\text{time}}$$

$$= \frac{(-5\mathbf{i} + 13\mathbf{j}) - (4\mathbf{i} + \mathbf{j})}{3}$$

$$= \frac{-9\mathbf{i} + 12\mathbf{j}}{3} = -3\mathbf{i} + 4\mathbf{j} \,\text{ms}^{-1}$$

(b)




The angle we want is $180 - \alpha$ and we see from above that $\alpha = \tan^{-1}\left(\frac{4}{3}\right) = 53.1^\circ$ .

So the angle we want is $127^\circ$ .

(c) It is at B with position vector $-5\mathbf{i} + 13\mathbf{j}$ m and then two second later it reaches C.

In those two seconds it has travelled $2 \times (-3\mathbf{i} + 4\mathbf{j}\,) = -6\mathbf{i} + 8\mathbf{j}$ m and so it is at the point with position vector $-5\mathbf{i} + 13\mathbf{j} + -6\mathbf{i} + 8\mathbf{j} = -11\mathbf{i} + 21\mathbf{j}$.

The distance OC is simply the magnitude of the position vector, so $\sqrt{(-11)^2 + 21^2} = 23.7$ m.

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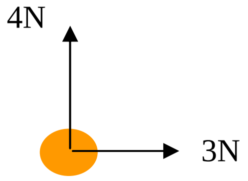
# Forces and Equilibrium

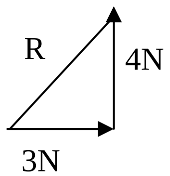
*Forces treated as vectors. Resolution of forces. Equilibrium of a particle under coplanar forces.*

*RESULTANT FORCE*
The resultant of a set forces is the one single force which is equivalent to that set of forces.

e.g. A horizontal force of 3N and a vertical force of 4N is applied to a body, what is the resultant force?

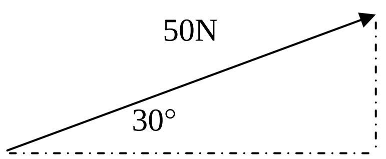



The resultant force, R, is the vector sum of these forces as shown below




So the resultant of these forces is (by Pythagoras' theorem) a force of 5N making angle of (by trigonometry) $\tan^{-1}\left(\frac{4}{3}\right) = 53.1°$ with the horizontal.

**COMPONENTS OF FORCES**
In the above we combined a horizontal and a vertical force to give a single force. We can also do the reverse. That is we could find the horizontal and vertical components of the 5N force in the above diagram. We will tend to resolve in two directions which are perpendicular to one another.
e.g. Find the horizontal and vertical components of a force of 50N which acts at an angle of $30°$ above the horizontal.




The horizontal component is $50\cos 30^\circ = 43.3\text{N}$ (to 3sf)
The vertical component is $50\sin 30^\circ = 25\text{N}$

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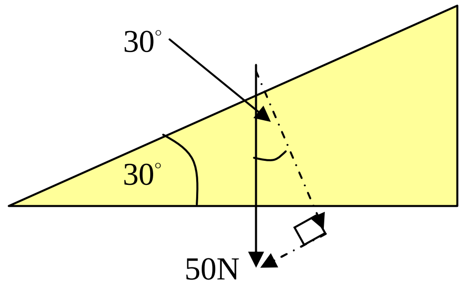
**Examples**

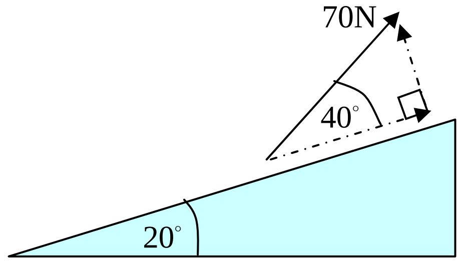
(a) Find the components parallel and perpendicular to a slope inclined at $30^\circ$ to the horizontal of a force of 50N acting vertically downwards.




The perpendicular component is the side of the right angled triangle that is adjacent to the angle of $30^\circ$.
Hence it is $50\cos 30^\circ = 43.3\,\text{N}$ (to 3sf).

The parallel component is the side of the right angled triangle that is opposite to the angle of $30^\circ$.
Hence it is $50\sin 30^\circ = 25\,\text{N}$.

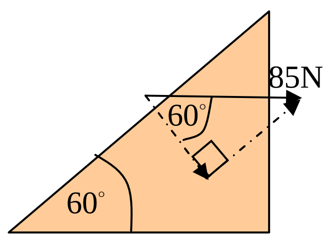
(b) Find the components parallel and perpendicular to a slope inclined at $20^\circ$ to the horizontal of a force of 70N acting at an angle of $60^\circ$ to the horizontal.




The perpendicular component is the side of the right angled triangle that is opposite to the angle of $40^\circ$.
Hence it is $70\sin 40^\circ = 45.0\,\text{N}$ (to 3sf).

The parallel component is the side of the right angled triangle that is adjacent to the angle of $40^\circ$.
Hence it is $70\cos 40^\circ = 53.6\,\text{N}$ (to 3sf).

(c) Find the components parallel and perpendicular to a slope inclined at $60^\circ$ to the horizontal of a force of 85N acting at an angle of $60^\circ$ to the horizontal.




The perpendicular component is the side of the right angled triangle that is adjacent to the angle of $60^\circ$.
Hence it is $85\cos 60^\circ = 42.5\,\text{N}$.

The parallel component is the side of the right angled triangle that is opposite to the angle of $60^\circ$.
Hence it is $85\sin 60^\circ = 73.6\,\text{N}$ (to 3sf).

**EQUILIBRIUM**

If a body is in equilibrium then the forces must balance each other, that is that the resultant force is zero. This means that the vector sum of the forces is zero and so we see that the sum of the components in any given direction is zero.

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*Weight, normal reaction, tension and thrust, friction. Only simple cases of the application of the conditions for equilibrium to uncomplicated systems will be required.*

**Weight** (often denoted by W) always acts vertically downwards. The weight (on the earth) of a mass of *m*kg is *mg* where $g = 9.8\text{ ms}^{-2}$.

So, for example, a 10kg block has weight 98N (on the earth).

**Normal Reaction** (often denoted by R) always acts in a direction that is perpendicular to the surface that the particle is lying on.

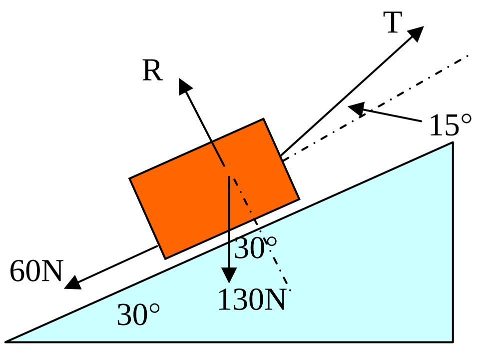
**Tension** (often denoted by T) always pulls on a particle.

**Thrust** (often denoted by T) always pushes on a particle.

**Friction** always opposes motion. So if the block is about to slip down the slope, friction must be acting up the slope.

**Example 1**

A block of weight 130N lies on a slope inclined at $30°$ to the horizontal. A string is attached to the block and is pulled at $45°$ to the horizontal in order to pull the block up the slope. The frictional force, *F*, is 60N when the block is just about to slip up the slope.
The tension is the string is *T* and the normal reaction force of the slope on the block is *R*.




Resolving parallel to the slope gives $T\cos 15^\circ - 130\sin 30^\circ - 60 = 0$ (1)

Resolving perpendicular to the slope gives $R + T\sin 15^\circ - 130\cos 30^\circ = 0$ (2)

Solving (1) gives $T = 129\text{N}$ (to 3sf)

Plugging this back into (2) gives $R = 79.1\text{N}$ (to 3sf)

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