Estimation - exam questions

Question 1: Jan 2010

Draught excluder for doors and windows is sold in rolls of nominal length 10 metres.

The actual length, X metres, of draught excluder on a roll may be modelled by a normal distribution with mean 10.2 and standard deviation 0.15.

(b) A customer randomly selects six 10-metre rolls of the draught excluder.

Calculate the probability that all six rolls selected contain more than 10 metres of draught excluder. *(3 marks)*

Question 2: Jan 2010

In a random sample of 12 bags of flour, the weight, in grams, of flour in each bag was recorded as follows.

1011	995	1018	1022	1014	1005	1017	1015	993	1018	992	1020

(a) It may be assumed that the weight of flour in a bag is normally distributed with a standard deviation of 10.5 grams.

(i) Construct a 98% confidence interval for the mean weight, μ grams, of flour in a bag, giving the limits to four significant figures. *(5 marks)*

(ii) State why, in constructing your confidence interval, use of the Central Limit Theorem was **not** necessary. *(1 mark)*

(iii) If the distribution of the weight of flour in a bag was unknown, indicate a minimum number of weights that you would consider necessary for a confidence interval for μ to be valid. *(1 mark)*

(b) The statement '1 kg' is printed on each bag.

Comment on this statement using **both** the confidence interval that you constructed in part (a)(i) and the weights of the given sample of 12 bags. *(3 marks)*

(c) Given that $\mu = 1000$, state the probability that a 98% confidence interval for μ will **not** contain 1000. *(1 mark)*

Question3: Jan 2009

UPVC facia board is supplied in lengths labelled as 5 metres. The actual length, X metres, of a board may be modelled by a normal distribution with a mean of 5.08 and a standard deviation of 0.05.

(b) Determine the probability that the mean length of a random sample of 4 boards:

(i) exceeds 5.05 metres; *(4 marks)*

(ii) is exactly 5 metres. *(1 mark)*

(c) Assuming that the value of the standard deviation remains unchanged, determine the mean length necessary to ensure that only 1 per cent of boards have lengths less than 5 metres. *(4 marks)*

Question 4: Jan 2009

The times taken by new recruits to complete an assault course may be modelled by a normal distribution with a standard deviation of 8 minutes.

A group of 30 new recruits takes a total time of 1620 minutes to complete the course.

(a) Calculate the mean time taken by these 30 new recruits. *(1 mark)*

(b) Assuming that the 30 recruits may be considered to be a random sample, construct a 98% confidence interval for the mean time taken by new recruits to complete the course. *(4 marks)*

(c) Construct an interval within which approximately 98% of the times taken by individual new recruits to complete the course will lie. *(2 marks)*

(d) State where, if at all, in this question you made use of the Central Limit Theorem. *(1 mark)*

Question 5: Jun 2006

The heights of sunflowers may be assumed to be normally distributed with a mean of 185 cm and a standard deviation of 10 cm.

(b) Determine the probability that the mean height of a random sample of 4 sunflowers is more than 190 cm. *(4 marks)*

Question 6: Jun 2006

The weights of packets of sultanas may be assumed to be normally distributed with a standard deviation of 6 grams.

The weights of a random sample of 10 packets were as follows:

498	496	499	511	503	505	510	509	513	508

(a) (i) Construct a 99% confidence interval for the mean weight of packets of sultanas, giving the limits to one decimal place. *(5 marks)*

(ii) State why, in calculating your confidence interval, use of the Central Limit Theorem was **not** necessary. *(1 mark)*

(iii) On each packet it states 'Contents 500 grams'.

Comment on this statement using **both** the given sample **and** your confidence interval. *(3 marks)*

(b) Given that the mean weight of all packets of sultanas is 500 grams, state the probability that a 99% confidence interval for the mean, calculated from a random sample of packets, will **not** contain 500 grams. *(1 mark)*

Question 7: Jun 2007

(a) A sample of 50 washed baking potatoes was selected at random from a large batch. The weights of the 50 potatoes were found to have a mean of 234 grams and a standard deviation of 25.1 grams.

Construct a 95% confidence interval for the mean weight of potatoes in the batch. *(4 marks)*

(b) The batch of potatoes is purchased by a market stallholder. He sells them to his customers by allowing them to choose any 5 potatoes for £1.

Give a reason why such chosen potatoes are unlikely to represent a random sample from the batch. *(1 mark)*

Question 8: Jun 2007

Electra is employed by E & G Ltd to install electricity meters in new houses on an estate. Her time, X minutes, to install a meter may be assumed to be normally distributed with a mean of 48 and a standard deviation of 20.

Gazali is employed by E & G Ltd to install gas meters in the same new houses. His time, Y minutes, to install a meter has a mean of 37 and a standard deviation of 25.

(i) Explain why Y is unlikely to be normally distributed. *(2 marks)*

(ii) State why $\overline{Y}$, the mean of a random sample of 35 gas meter installations, is likely to be approximately normally distributed. *(1 mark)*

(iii) Determine $P(\overline{Y} > 40)$. *(4 marks)*

Question 9: Jan 2007

A very popular play has been performed at a London theatre on each of 6 evenings per week for about a year. Over the past 13 weeks (78 performances), records have been kept of the proceeds from the sales of programmes at each performance. An analysis of these records has found that the mean was £184 and the standard deviation was £32.

(a) Assuming that the 78 performances may be considered to be a random sample, construct a 90% confidence interval for the mean proceeds from the sales of programmes at an evening performance of this play. *(4 marks)*

(b) Comment on the likely validity of the assumption in part (a) when constructing a confidence interval for the mean proceeds from the sales of programmes at an evening performance of:

(i) this particular play;

(ii) any play. *(3 marks)*

Question 10: Jun 2009

The weight, X grams, of talcum powder in a tin may be modelled by a normal distribution with mean 253 and standard deviation σ.

Assuming that the value of the mean remains unchanged, determine the value of σ necessary to ensure that 98% of tins contain more than 245 grams of talcum powder. *(4 marks)*

Estimation - Answers

Question 1: "Draught excluder"

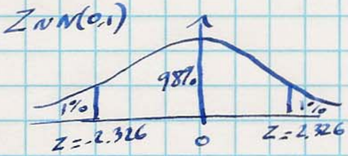
$X \sim N(10.2, 0.15^2)$

- $P(X>10) = P\left(Z > \frac{10.0-10.2}{0.15}\right) = P(Z>-1.33)$
  $= P(Z<1.33)$
  $= \boxed{0.908}$

- $P(6 \text{ rolls} > 10) = 0.908^6 = \boxed{0.56}$

Question 2: "Bags of flour"

- mean weight: $\boxed{1010}$
- $\bar{X} \sim N\left(1010, \left(\frac{10.5}{\sqrt{12}}\right)^2\right)$ Standard error: $\frac{\sigma}{\sqrt{n}} = 3.031$



i) 98% confidence interval

$\bar{x} - Z \times \frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + Z \times \frac{\sigma}{\sqrt{n}}$

$1010 - 2.326 \times 3.031 < \mu < 1010 + 2.326 \times 3.031$

$\boxed{1003 < \mu < 1017}$

ii) Because the weight of flour is assumed to be normally distributed.

iii) 30 or more

b) The confidence interval (1003-1017) is more than 1000 AND only 3 out 10 bags in the sample is less than 1000.
So the indication 1kg is conservative.

c) 2% of course!

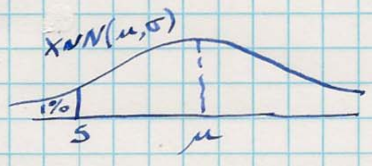
Question 3 "UPVC boards"

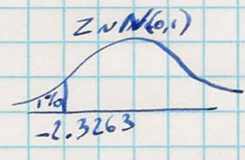
$\bar{X} \sim N\left(5.08, \left(\frac{0.05}{\sqrt{4}}\right)^2\right)$ Standard error: $\frac{\sigma}{\sqrt{n}} = 0.025$

i) $P(\bar{X} > 5.05) = P\left(Z > \frac{5.05-5.08}{0.025}\right)$
$= P(Z>-1.2) = P(Z<1.2) = \boxed{0.885}$

ii) $P(\bar{X} = 5) = 0$

c)





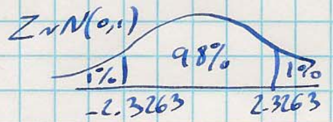
$x = \mu + Z\sigma$
$5 = \mu - 2.3263 \times 0.05$

$\boxed{\mu = 5.12}$

Question 4: "Assault course"

a) $\text{mean} = \frac{1620}{30} = \boxed{54 \text{ min}}$

b) $\bar{X} \sim N\left(54, \left(\frac{8}{\sqrt{30}}\right)^2\right)$ Standard error: $\frac{\sigma}{\sqrt{n}} = 1.4606$



98% confidence interval

$\bar{x} - 2.3263\frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + 2.3263\frac{\sigma}{\sqrt{n}}$

$54 - 2.3263 \times 1.4606 < \mu < 54 + 2.3263 \times 1.4606$

$\boxed{50.6 < \mu < 57.4}$

c) $\mu \pm 2.3263\sigma$

$54 \pm 2.3263 \times 8$

$\boxed{35.4 \text{ min} \Leftrightarrow 72.6 \text{ min}}$

d) No because the time taken is assumed to be normally distributed.

Question 5 "Sunflowers"

$\bar{X} \sim N\left(185, \left(\frac{10}{\sqrt{4}}\right)^2\right)$ Standard error $\frac{\sigma}{\sqrt{n}} = \frac{10}{2} = 5$

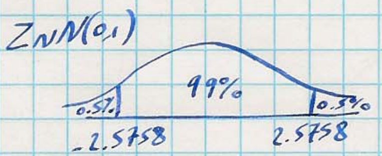
$P(\bar{X} > 190) = P\left(Z > \frac{190-185}{5}\right) = P(Z > 1)$

$= 1 - P(Z < 1) = \boxed{0.159}$

Question 6 "Packets of sultanas"

a) i). mean: $\boxed{505.2 \text{ g}}$

$\bar{X} \sim N\left(505.2, \left(\frac{6}{\sqrt{10}}\right)^2\right)$ Standard error: $\frac{\sigma}{\sqrt{n}} = 1.8974$



99% confidence interval

$\bar{x} - 2.5758\frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + 2.5758\frac{\sigma}{\sqrt{n}}$

$505.2 - 2.5758 \times 1.8974 < \mu < 505.2 + 2.5758 \times 1.8974$

$\boxed{500.3 < \mu < 510.1}$

ii) the weight is assumed to be Normally distributed.

iii) The confidence interval is above 500g and only 3 out 10 packets in the sample is below 500g.

b) 1% of course!

Question 7: June 2007

(a)	95% $\Rightarrow$ z = 1.96 or 95% $\Rightarrow$ t = 2.0 to 2.01 (Knowledge of the t–distribution is not required in this unit)	B1 (B1)	
	CI for μ is $\bar{x} \pm (z \text{ or } t) \times \frac{(s_{n-1} \text{ or } s_n)}{\sqrt{n}}$	M1	
	Note that $25.1 \times \sqrt{\frac{50}{49}} = 25.35483$		
	Thus $234 \pm (1.96 \text{ or } 2.009) \times \frac{(25.1 \text{ or } 25.3 \text{ to } 25.4)}{(\sqrt{50} \text{ or } \sqrt{49})}$	A1√	
	Hence 234 ± (6.95 to 7.30)		
	ie 234 ± 7 or (227, 241)	A1	4
(b)	Customers are likely to choose large / similar sized potatoes	B1	1
	Total		**5**

Question 8: June 2007

(a)	Time, $X \sim N(48, 20^2)$		
(i)	$P(X < 60) = P\left(Z < \frac{60-48}{20}\right) =$	M1	
	$P(Z < 0.6)$ = 0.725 to 0.73	A1	2
(ii)	$P(30 < X < 60) =$ $P(X < 60) - P(X < 30) =$ (i) $- P(X < 30) =$ (i) $- P(Z < -0.9) =$	M1	
	(i) $- \{1 - P(Z < +0.9)\} =$ $0.72575 - \{1 - 0.81594\} =$	m1	
	0.54 to 0.542	A1	3
iii)	$0.9 \Rightarrow z = 1.28$ to 1.282	B1	
	$z = \frac{k-48}{20}$	M1	
	= 1.2816	m1	
	k = 73.6 to 74	A1	4
(b)	Time, $Y \sim N(37, 25^2)$		
(i)	Use of $\mu - (2 \text{ or } 3) \times \sigma =$ 37 – (50 or 75)	M1	
	$< 0 \Rightarrow$ likely negative times	B1	2
(ii)	Central Limit Theorem or n large / > 30	B1	1
iii)	Variance of $\bar{Y}$ = $\frac{25^2}{35}$	B1	
	$P(\bar{Y} > 40) = P\left(Z > \frac{40-37}{25/\sqrt{35}}\right) =$	M1	
	$P(Z > 0.71) = 1 - P(Z < 0.71) =$	m1	
	0.238 to 0.24	A1	4
	Total		**16**

Question 9: Jan2007

4(a)	90% $\Rightarrow$ z = 1.64 to 1.65 or 90% $\Rightarrow$ t = 1.66 to 1.67 (Knowledge of the t–distribution is **not** required in this unit)	B1 (B1)	
	CI for μ is $\bar{x} \pm (z \text{ or } t) \times \frac{(s_{n-1} \text{ or } s_n)}{\sqrt{n}}$	M1	
	Thus $184 \pm (1.6449 \text{ or } 1.6649) \times \frac{(32 \text{ or } 32.2)}{(\sqrt{78} \text{ or } \sqrt{77})}$	A1√	
	Hence 184 ± (5.94 to 6.13)		
	or £184 ± £6		
	or (£178, £190)	A1	4
b)(i)	**Likely to be valid**	B1	
(ii)	Different plays have different: programme prices, sales, marketing, etc theatre or audience sizes, etc popularity, artists, etc so **Unlikely to be valid**	B1 ↑Dep↑ B1	3
	Total		**7**

Question 10: June 2009

(b)	98% (0.98) $\Rightarrow$ z = –2.05 to –2.06	B1	
	$z = \frac{245-253}{\sigma}$	M1	
	= –2.0537	A1	
	σ = 3.88 to 3.9(0)	A1	
	Note: $\frac{245-253}{\sigma} = 2.0537 \Rightarrow \sigma = 3.8954$ $\Rightarrow$ B1 M1 A1 A0		4