SMP 16-19 Mathematics – Revision Notes
Unit 2 – Introductory Calculus

Rates Of Change

1. For a function, $y = f(x)$, then the gradient of that function is given as $\frac{dy}{dx}$.
2. For a linear function in the form $y = mx + c$ then $\frac{dy}{dx} = m$.
3. If u and v are linear functions of x, and a and b are constants, then $y = au + bv$ is also a linear function of x and:

$$\frac{dy}{dx} = a\frac{du}{dx} + b\frac{dv}{dx}$$

Gradients Of Curves

1. If a curve appears to be linear when you zoom in at a point, then it is 'locally straight' at that point.
2. The gradient of a curve at a point is equal to the gradient of the tangent to the curve at that point.
3. Differentiation is the process of obtaining $\frac{dy}{dx}$ for a given function $y = f(x)$.
4. Points on a graph can be given names:
   a. Stationary points – where the curve has zero gradient.
   b. Turning points – where the graph is a local maximum or minimum.
   c. Points of inflection – where the gradient graph is a turning point and a stationary point.
5. In order to sketch a gradient graph, find the stationary points where the gradient is 0, and then look to see whether it is positive or negative along the curve.
6. The gradient of a function $f(x)$ at a point $(a, f(a))$ can be estimated numerically by taking a small change in x (δx):

$$f'(a) \approx \frac{f(a+\delta x) - f(a)}{\delta x}$$

7. For a polynomial $y = a + bx + cx^2 + dx^3$, then $\frac{dy}{dx} = b + 2cx + 3dx^2$.
8. Leibnitz notation states that the gradient of the function $f(x)$ can be written as $\frac{d}{dx}(f(x))$.

Optimisation

1. The gradient of a graph at appoint tells you what the gradient is like near that point.
2. Graphs of quadratics and cubics can be quickly sketched by:
   a. Finding the y-intercept.
   b. Considering the sign of the highest power of x to determine the shape for large $|x|$.
   c. Finding the x-coordinates of any stationary points by solving $\frac{dy}{dx} = 0$.
3. Calculus can be used to find the local maximum and minimum values of a quantity, by expressing the quantity as an equation in terms of another variable. Calculus can then be used to find maxima and minima for the equation, by solving $\frac{dy}{dx} = 0$. This is the process of optimisation.

Numerical Integration

1. The area under a graph will represent a quantity, depending on the quantities of the axes. This is a definite integral.
2. The precise value of the area underneath a graph of $y = f(x)$, between $x = a$ and $x = b$ is shown as:

$$\int_a^b f(x)\,dx$$

3. The mid-ordinate rule uses a series of rectangles, with height of the midpoint of the curve, to estimate the area under the graph. This is shown as:

$$\int_a^b f(x)\,dx \approx \sum_{r=1}^{n} hy_r$$
$$h = \frac{b-a}{n}$$
$$x_1 = a + \tfrac{1}{2}h$$
$$x_{r+1} = x_r + h$$
$$y_r = f(x_r)$$

4. The trapezium rule uses a series of trapezia, joining point on the curve, to estimate the area under the graph. This is represented as:

$$\int_a^b f(x)\,dx \approx \sum_{r=1}^{n} \tfrac{1}{2}h\left(y_{r-1} + y_r\right)$$
$$h = \frac{b-a}{n}$$
$$x_0 = a$$
$$x_{r+1} = x_r + h$$
$$y_r = f(x_r)$$

5. For a parabola-type curve, then the trapezium rule will give an overestimate and the mid-ordinate rule will give an underestimate of the area under the curve.
6. For an inverted parabola-type curve, then the trapezium rule will give an underestimate and the mid-ordinate rule will give an overestimate of the area under the curve.
7. Integrals are defined to give negative areas below the *x*-axis. This is not wanted when calculating areas though, so:

$$\int_a^b y\,dx = A - B$$

Algebraic Integration

1. Any function $f(x)$ can have the function of its area expressed as $A(x)$.
2. For the polynomial $f(x) = a + bx + cx^2 + dx^3$ then $A(x) = ax + \tfrac{1}{2}bx^2 + \tfrac{1}{3}cx^3 + \tfrac{1}{4}dx^4$.
3. The fundamental theorem of calculus states that $\frac{d}{dx}\left(A(x)\right) = f(x)$.
4. For any differentiable function, *f*, then:

$$\int_a^b f'(x)\,dx = f(b) - f(a)$$

5. An indefinite integral is one without limits, and must always have a constant term, *c*, included. This is the constant of integration. In definite integrals this constant simply cancels out.