Binomial distribution - exam questions

Question 1: Jun 2009 Q7

Mr Alott and Miss Fewer work in a postal sorting office.

(a) The number of letters per batch, R, sorted incorrectly by Mr Alott when sorting batches of 50 letters may be modelled by the distribution $\text{B}(50,\ 0.15)$.

Determine:

(i) $\text{P}(R < 10)$;

(ii) $\text{P}(5 \leqslant R \leqslant 10)$. *(4 marks)*

Question 2: Jan 2011 Q4

Clay pigeon shooting is the sport of shooting at special flying clay targets with a shotgun.

(a) Rhys, a novice, uses a single-barrelled shotgun. The probability that he hits a target is 0.45, and may be assumed to be independent from target to target.

Determine the probability that, in a series of shots at 15 targets, he hits:

(i) at most 5 targets; *(1 mark)*

(ii) more than 10 targets; *(2 marks)*

(iii) exactly 6 targets; *(2 marks)*

(iv) at least 5 but at most 10 targets. *(3 marks)*

(b) Sasha, an expert, uses a double-barrelled shotgun. She shoots at each target with the gun's first barrel and, only if she misses, does she then shoot at the target with the gun's second barrel.

The probability that she hits a target with a shot using her gun's first barrel is 0.85. The conditional probability that she hits a target with a shot using her gun's second barrel, given that she has missed the target with a shot using her gun's first barrel, is 0.80. Assume that Sasha's shooting is independent from target to target.

(i) Show that the probability that Sasha hits a target is 0.97. *(2 marks)*

(ii) Determine the probability that, in a series of shots at 50 targets, Sasha hits at least 48 targets. *(3 marks)*

(iii) In a series of shots at 80 targets, calculate the mean number of times that Sasha shoots at targets with her gun's second barrel. *(2 marks)*

Question 3: Jan 2010 Q6

During the winter, the probability that Barry's cat, Sylvester, chooses to stay outside all night is 0.35, and the cat's choice is independent from night to night.

(a) Determine the probability that, during a period of 2 weeks (14 nights) in winter, Sylvester chooses to stay outside:

(i) on at most 7 nights; *(2 marks)*

(ii) on at least 11 nights; *(2 marks)*

(iii) on more than 5 nights but fewer than 10 nights. *(3 marks)*

(b) Calculate the probability that, during a period of **3 weeks** in winter, Sylvester chooses to stay outside on exactly 4 nights. *(3 marks)*

(c) Barry claims that, during the summer, the number of nights per week, S, on which Sylvester chooses to stay outside can be modelled by a binomial distribution with $n = 7$ and $p = \frac{5}{7}$.

(i) Assuming that Barry's claim is correct, find the mean and the variance of S. *(2 marks)*

(ii) For a period of 13 weeks during the summer, the number of nights per week on which Sylvester chose to stay outside had a mean of 5 and a variance of 1.5.

Comment on Barry's claim. *(2 marks)*

Question 4: Jan 2007 Q2

A hotel has 50 single rooms, 16 of which are on the ground floor. The hotel offers guests a choice of a full English breakfast, a continental breakfast or no breakfast. The probabilities of these choices being made are 0.45, 0.25 and 0.30 respectively. It may be assumed that the choice of breakfast is independent from guest to guest.

(a) On a particular morning there are 16 guests, each occupying a single room on the ground floor. Calculate the probability that exactly 5 of these guests require a full English breakfast. *(3 marks)*

(b) On a particular morning when there are 50 guests, each occupying a single room, determine the probability that:

(i) at most 12 of these guests require a continental breakfast; *(2 marks)*

(ii) more than 10 but fewer than 20 of these guests require no breakfast. *(3 marks)*

(c) When there are 40 guests, each occupying a single room, calculate the mean and the standard deviation for the number of these guests requiring breakfast. *(4 marks)*

Binomial distribution - exam questions MS

Question 1: Jun 2009

(a)	$R \sim B(50,\ 0.15)$		
(i)	$P(R < 10) = \mathbf{0.791}$	B1	
(ii)	$P(5 \le R \le 10) = 0.8801$ or 0.7911 (p_1)	M1	
	minus 0.1121 or 0.2194 (p_2)	M1	
	$= 0.768$	A1	
	or		
	B(50, 0.15) expressions stated for **at least 3** terms within $4 \le R \le 10$ gives probability	(M1)	
	$= 0.768$	(A2)	4

Question 2: Jan 2011

4(a)	$R \sim B(15, 0.45)$		
(i)	$P(R \le 5) = 0.26(0)$ to 0.261	B1	1
(ii)	$P(R > 10) = 1 - P(R \le 10)$		
	$= 1 - (0.9745$ or $0.9231)$	M1	
	$= 0.025$ to 0.026	A1	2
(iii)	$P(R = 6) = 0.4522 - (a)(i)$		
	or $= \binom{15}{6}(0.45)^6(0.55)^9$	M1	
	$= 0.191$ to 0.192	A1	2
(iv)	$P(5 \le R \le 10) = 0.9745$ or 0.9231 (p_1)	M1	
	Minus 0.1204 or 0.2608 (p_2)	M1	
	$= 0.853$ to 0.855	A1	3
	Or B (15, 0.45) terms stated for at least 3 values within $4 \le R \le 11$ gives probability	(M1)	
	$= 0.853$ to 0.855	(A2)	
(b)(i)	$P(S) = 0.85$ plus 1 minus	B1	
	(0.15×0.80) (0.15×0.20)	B1	2
	$= 0.97$		
	NB: $(0.85 \times 0.20) + 0.80 \Rightarrow$ B0 B0 $(0.85 \times 0.20) + (0.85 \times 0.80) + (0.15 \times 0.80) \Rightarrow$ B0 B1		
(ii)	$P(S \ge 48) = 0.81$ to 0.82 or 0.5553 or 0.9372	M2	
	$= 0.81(0)$ to 0.811	A1	3
	NB: Answer $= 0.4447$ or 0.1892 or $0.0628 \Rightarrow$ M1 only		
(iii)	$p = 1 - 0.85 = 0.15$	B1	
	Mean, $\mu = 80 \times 0.15 = 12$	B1	2
	SC Mean $= 9.6 \Rightarrow$ B1 only		
	Total		**15**

Question 3: Jan 2010

(a)(i)	$R \sim B(14,\ 0.35)$	M1	
	$P(R \le 7) = 0.924$ to 0.925	A1	2
(ii)	$P(R \ge 11) = 1 - P(R \le 10)$		
	$= 1 - (0.9989$ or $0.9999)$	M1	
	$= 0.0011$	A1	2
(iii)	$P(5 < R < 10) = 0.9940$ or 0.9989 (p_1)	M1	
	minus 0.6405 or 0.4227 (p_2)	M1	
	$= 0.353$ to 0.354	A1	3
	or B(14, 0.35) expressions stated for **at least 3** terms within $4 \le R \le 11$ gives probability	(M1)	
	$= 0.353$ to 0.354	(A2)	
(b)	$R \sim B(21,\ 0.35)$	M1	
	$P(R = 4) = \binom{21}{4}(0.35)^4(0.65)^{17}$	A1	
	$= 0.059$ to 0.0595	A1	3
(c)(i)	$S \sim B(7,\ 5/7)$ Mean $= np = 7 \times 5/7 = 5$	B1	
	If not identified, assume order is μ then σ^2		
	Variance $= np(1-p)$ $= 7 \times 5/7 \times 2/7 = 10/7$ or 1.42 to 1.43	B1	2
(ii)	**Means are the same** **and (both comparisons clearly stated)** **Variances/standard deviations are similar** Do not accept statements involving correct/incorrect/exact/etc	B1dep	
	Barry's claim appears/is **sound/valid/correct/likely**	B1dep	2
		Total	14

Question 4: Jan 2007

2(a)	Use of binomial in (a), (b) or (c)	M1	
	$P(E = 5) = \binom{16}{5}(p)^5(1-p)^{11}$	M1	
	$= 0.112$	A1	3
(b)(i)	B(50, 0.25)	B1	
	$P(C \le 12) = 0.511$	B1	2
(ii)	$P(10 < B' < 20) = 0.9152$ or 0.9522	M1	
	minus 0.0789 or 0.1390	M1	
	$= 0.836$	A1	3
	or B(50, 0.30) expressions stated for **at least 3** terms within $10 \le B' \le 20$	(M1)	
	Answer $= 0.836$	(A2)	
(c)	$n = 40,\ p = 0.7$	B1	
	Mean $\mu = np = 28$	B1√	
	Variance $\sigma^2 = np(1-p) = 8.4$	M1	
	Standard deviation $= \sqrt{8.4}$ or $= 2.89$ to 2.9	A1	4
	Total		**12**