## Monday 28 January 2013 - Morning

## A2 GCE MATHEMATICS (MEI)

4758/01 Differential Equations

## QUESTION PAPER

Candidates answer on the Printed Answer Book.
OCR supplied materials:
Duration: 1 hour 30 minutes

- Printed Answer Book 4758/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:

- Scientific or graphical calculator


## INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- Write your answer to each question in the space provided in the Printed Answer Book. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer any three questions.
- Do not write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $\mathrm{gm} \mathrm{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g=9.8$.


## INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive no marks unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is 72.
- The Printed Answer Book consists of 16 pages. The Question Paper consists of 4 pages. Any blank pages are indicated.


## INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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1 The differential equation

$$
\frac{\mathrm{d}^{3} y}{\mathrm{~d} x^{3}}+2 \frac{\mathrm{~d}^{2} y}{\mathrm{~d} x^{2}}-5 \frac{\mathrm{~d} y}{\mathrm{~d} x}-6 y=\sin x
$$

is to be solved.
(i) Show that 2 is a root of the auxiliary equation. Find the other two roots and hence find the general solution of the differential equation.
When $x=0, y=1$ and $\frac{\mathrm{d} y}{\mathrm{~d} x}=0$. Also, $y$ is bounded as $x \rightarrow \infty$.
(ii) Find the particular solution.
(iii) Write down an approximate solution for large positive values of $x$. Calculate the amplitude of this approximate solution and sketch the solution curve for large positive $x$.

Suppose instead that a solution is required that is bounded as $x \rightarrow-\infty$.
(iv) Determine whether there is a solution for which $y=1$ and $\frac{\mathrm{d} y}{\mathrm{~d} x}=0$ when $x=0$.

2 A ball of mass $m \mathrm{~kg}$ falls vertically from rest through a liquid. At time $t \mathrm{~s}$, the velocity of the ball is $v \mathrm{~ms}^{-1}$ and the ball has fallen a distance $x \mathrm{~m}$. The forces on the ball are its weight and a total upwards force of $R \mathrm{~N}$. A student investigates three models for $R$.

In the first model $R=m k v$, where $k$ is a positive constant.
(i) Show that $\frac{\mathrm{d} v}{\mathrm{~d} t}=9.8-k v$ and hence find $v$ in terms of $t$ and $k$.

The terminal velocity of the ball is observed to be $7 \mathrm{~m} \mathrm{~s}^{-1}$.
(ii) Find $k$.

In the second model, $R=0.2 m v^{2}$.
(iii) Find $v$ in terms of $t$. Show that your solution is consistent with a terminal velocity of $7 \mathrm{~ms}^{-1}$.

In the third model, $R=0.529 m v^{\frac{3}{2}}$. Euler's method is to be used to solve for $v$ numerically.
The algorithm is given by $t_{r+1}=t_{r}+h, v_{r+1}=v_{r}+h \dot{v}_{r}$ with $\left(t_{0}, v_{0}\right)=(0,0)$.
(iv) Show that $\frac{\mathrm{d} v}{\mathrm{~d} t}=9.8-0.529 v^{\frac{3}{2}}$ and find $v$ when $t=0.2$ using Euler's method with a step length of 0.1.
(v) Show that this model is consistent with a terminal velocity of approximately $7 \mathrm{~ms}^{-1}$.

3 (a) Solve the differential equation

$$
\begin{equation*}
\frac{\mathrm{d} y}{\mathrm{~d} x}-y \tan x=\sin x \tag{9}
\end{equation*}
$$

to find $y$ in terms of $x$ subject to the condition $y=1$ when $x=0$.
(b) Consider the differential equations

$$
\begin{align*}
& \frac{\mathrm{d} y}{\mathrm{~d} x}+\mathrm{f}(x) y=\mathrm{g}(x)  \tag{1}\\
& \frac{\mathrm{d} y}{\mathrm{~d} x}+\mathrm{f}(x) y=0 \tag{2}
\end{align*}
$$

Show that if $y=\mathrm{p}(x)$ satisfies (1) and $y=\mathrm{c}(x)$ satisfies (2), then $y=\mathrm{p}(x)+A \mathrm{c}(x)$ satisfies (1), where $A$ is an arbitrary constant.
(c) The differential equation

$$
\begin{equation*}
\frac{\mathrm{d} y}{\mathrm{~d} x}+\frac{2 y}{x}=2 \mathrm{e}^{x^{2}}\left(\frac{x^{2}+1}{x}\right) \tag{3}
\end{equation*}
$$

is to be solved.
(i) Verify that $y=\mathrm{e}^{x^{2}}$ satisfies (3).
(ii) Find the general solution of $\frac{\mathrm{d} y}{\mathrm{~d} x}+\frac{2 y}{x}=0$, giving $y$ in terms of $x$.
(iii) Use the result of part (b) to find a solution of (3) for which $y=1$ when $x=1$.

4 The simultaneous differential equations

$$
\begin{aligned}
& \frac{\mathrm{d} x}{\mathrm{~d} t}=-\frac{1}{2} x-\frac{3}{2} y+t \\
& \frac{\mathrm{~d} y}{\mathrm{~d} t}=\frac{3}{2} x-\frac{1}{2} y+2 t
\end{aligned}
$$

are to be solved.
(i) Eliminate $y$ to obtain a second order differential equation for $x$ in terms of $t$. Hence find the general solution for $x$.
(ii) Find the corresponding general solution for $y$.

When $t=0, x=1$ and $y=0$.
(iii) Find the particular solutions.
(iv) Show that in this case $x+y$ tends to a finite limit as $t \rightarrow \infty$ and state its value. Determine whether $x+y$ is equal to this limit for any values of $t$.

THERE ARE NO QUESTIONS PRINTED ON THIS PAGE.

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