

# Friday 1 June 2012 – Morning

## A2 GCE MATHEMATICS (MEI)

4764 Mechanics 4

### QUESTION PAPER



Candidates answer on the Printed Answer Book.

**OCR supplied materials:**

- Printed Answer Book 4764
- MEI Examination Formulae and Tables (MF2)

**Other materials required:**

- Scientific or graphical calculator

**Duration:** 1 hour 30 minutes

### INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by  $gm s^{-2}$ . Unless otherwise instructed, when a numerical value is needed, use  $g = 9.8$ .

### INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

### INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

## Section A (24 marks)

- 1 A rocket in deep space has initial mass  $m_0$  and is moving in a straight line at speed  $v_0$ . It fires its engine in the direction opposite to the motion in order to increase its speed. The propulsion system ejects matter at a constant mass rate  $k$  with constant speed  $u$  relative to the rocket. At time  $t$  after the engines are fired, the speed of the rocket is  $v$ .

(i) Show that while mass is being ejected from the rocket,  $(m_0 - kt) \frac{dv}{dt} = uk$ . [6]

(ii) Hence find an expression for  $v$  at time  $t$ . [5]

- 2 A light elastic string AB has stiffness  $k$ . The end A is attached to a fixed point and a particle of mass  $m$  is attached at the end B. With the string vertical, the particle is released from rest from a point at a distance  $a$  below its equilibrium position. At time  $t$ , the displacement of the particle below the equilibrium position is  $x$  and the velocity of the particle is  $v$ .

(i) Show that

$$mv \frac{dv}{dx} = -kx. \quad [4]$$

(ii) Show that, while the particle is moving upwards and the string is taut,

$$v = -\sqrt{\frac{k}{m}(a^2 - x^2)}. \quad [5]$$

(iii) Hence use integration to find an expression for  $x$  at time  $t$  while the particle is moving upwards and the string is taut. [4]

## Section B (48 marks)

- 3 A uniform rigid rod AB of length  $2a$  and mass  $m$  is smoothly hinged to a fixed point at A so that it can rotate freely in a vertical plane. A light elastic string of modulus  $\lambda$  and natural length  $a$  connects the midpoint of AB to a fixed point C which is vertically above A with  $AC = a$ . The rod makes an angle  $2\theta$  with the upward vertical, where  $\frac{1}{3}\pi \leq 2\theta \leq \pi$ . This is shown in Fig. 3.

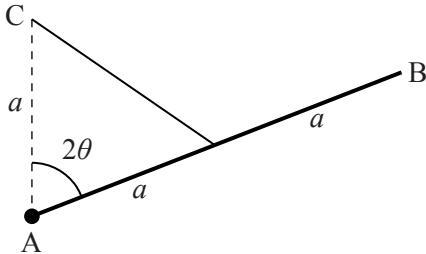


Fig. 3

- (i) Find the potential energy,  $V$ , of the system relative to A in terms of  $m$ ,  $\lambda$ ,  $a$  and  $\theta$ . Show that

$$\frac{dV}{d\theta} = 2a \cos \theta (2\lambda \sin \theta - 2mg \sin \theta - \lambda). \quad (*) \quad [7]$$

Assume now that the system is set up so that the result (\*) continues to hold when  $\pi < 2\theta \leq \frac{5}{3}\pi$ .

- (ii) In the case  $\lambda < 2mg$ , show that there is a stable position of equilibrium at  $\theta = \frac{1}{2}\pi$ . Show that there are no other positions of equilibrium in this case. [9]

- (iii) In the case  $\lambda > 2mg$ , find the positions of equilibrium for  $\frac{1}{3}\pi \leq 2\theta \leq \frac{5}{3}\pi$  and determine for each whether the equilibrium is stable or unstable, justifying your conclusions. [7]

- 4 (i) Show by integration that the moment of inertia of a uniform circular lamina of radius  $a$  and mass  $m$  about an axis perpendicular to the plane of the lamina and through its centre is  $\frac{1}{2}ma^2$ . [6]

A closed hollow cylinder has its curved surface and both ends made from the same uniform material. It has mass  $M$ , radius  $a$  and height  $h$ .

- (ii) Show that the moment of inertia of the cylinder about its axis of symmetry is  $\frac{1}{2}Ma^2\left(\frac{a+2h}{a+h}\right)$ . [6]

For the rest of this question take the cylinder to have mass 8 kg, radius 0.5 m and height 0.3 m.

The cylinder is at rest and can rotate freely about its axis of symmetry. It is given a tangential impulse of magnitude 55 N s at a point on its curved surface. The impulse is perpendicular to the axis.

- (iii) Find the angular speed of the cylinder after the impulse. [3]

A resistive couple is now applied to the cylinder for 5 seconds. The magnitude of the couple is  $2\dot{\theta}^2$  N m, where  $\dot{\theta}$  is the angular speed of the cylinder in  $\text{rad s}^{-1}$ .

- (iv) Formulate a differential equation for  $\dot{\theta}$  and hence find the angular speed of the cylinder at the end of the 5 seconds. [7]

The cylinder is now brought to rest by a constant couple of magnitude 0.03 N m.

- (v) Calculate the time it takes from when this couple is applied for the cylinder to come to rest. [3]



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## **Friday 1 June 2012 – Morning**

### **A2 GCE MATHEMATICS (MEI)**

**4764      Mechanics 4**

#### **PRINTED ANSWER BOOK**



Candidates answer on this Printed Answer Book.

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- Question Paper 4764 (inserted)
- MEI Examination Formulae and Tables (MF2)

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- Scientific or graphical calculator

**Duration: 1 hour 30 minutes**



Candidate forename		Candidate surname	
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Centre number						Candidate number			
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## **Section A (24 marks)**

**1(i)**

**1(ii)**

<b>2(i)</b>	
<b>2(ii)</b>	
	<b>(answer space continued on next page)</b>

2(ii)	(continued)
<hr/>	
2(iii)	
<hr/>	

**Section B (48 marks)**

**3(ii)**



**4(i)**

<b>4(ii)</b>	
<hr/> <b>4(iii)</b>	



**4(v)**

## **Mathematics (MEI)**

Advanced GCE

Unit **4764**: Mechanics 4

# **Mark Scheme for June 2012**

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This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

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**Annotations**

<b>Annotation in scores</b>	<b>Meaning</b>
✓ and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
<b>Other abbreviations in mark scheme</b>	<b>Meaning</b>
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

**Subject-specific Marking Instructions**

- a Annotations should be used whenever appropriate during your marking.

**The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks.** It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

- b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

- c The following types of marks are available.

**M**

A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

**A**

Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

**B**

Mark for a correct result or statement independent of Method marks.

**E**

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

- d When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep \*’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.
- e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be ‘follow through’. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

- f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.)

We are usually quite flexible about the accuracy to which the final answer is expressed and we do not penalise over-specification.

#### **When a value is given in the paper**

Only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case.

**When a value is not given in the paper**

Accept any answer that agrees with the correct value to 2 s.f.

It should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination.

There is no penalty for using a wrong value for  $g$ . E marks will be lost except when results agree to the accuracy required in the question.

- g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

- h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working.

'Fresh starts' will not affect an earlier decision about a misread.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

- i If a graphical calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.
- j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question		Answer	Marks	Guidance
1	(i)	$mv = (m + \delta m)(v + \delta v) + (-\delta m)(v - u) \quad (\text{note } \delta m < 0)$ $mv = mv + v\delta m + m\delta v + \delta m\delta v - v\delta m + u\delta m$ $m \frac{\delta v}{\delta t} + u \frac{\delta m}{\delta t} + \delta m \frac{\delta v}{\delta t} = 0$ $m \frac{dv}{dt} = -u \frac{dm}{dt}$ $\frac{dm}{dt} = -k$ $m = m_0 - kt$ $(m_0 - kt) \frac{dv}{dt} = uk$	M1 A1 M1 B1 B1 E1 <b>[6]</b>	Attempt at momentum equation Condone wrong sign of $\delta m$ Simplify and divide by $\delta t$ SOI All correct including sign of $\delta m$
1	(ii)	$\int dv = \int \frac{uk}{m_0 - kt} dt$ $v = -u \ln(m_0 - kt) + c$ $t = 0, v = v_0 \Rightarrow v_0 = -u \ln m_0 + c$ $c = v_0 + u \ln m_0$ $v = v_0 + u \ln \left( \frac{m_0}{m_0 - kt} \right)$	M1 A1 M1 A1 A1 <b>[5]</b>	Separate and integrate Use condition aef
2	(i)	Let equilibrium extension be $e$ $mv \frac{dv}{dx} = mg - k(e + x)$ At equilibrium, $mg = ke$ So $mv \frac{dv}{dx} = -kx$	M1 A1 B1 E1 <b>[4]</b>	N2L All terms correct oe With evidence of working

Question		Answer	Marks	Guidance
2	(ii)	$\int mv dv = \int -kx dx$ $\frac{1}{2}mv^2 = -\frac{1}{2}kx^2 + c$ $x = a, v = 0 \Rightarrow 0 = -\frac{1}{2}ka^2 + c$ $\frac{1}{2}mv^2 = \frac{1}{2}k(a^2 - x^2)$ $v = -\sqrt{\frac{k}{m}(a^2 - x^2)}$ $(v < 0 \text{ when moving up})$	M1 A1 M1 A1 E1 [5]	Solutions from SHM acceptable  oe  AG Complete argument including justification for square root.
2	(iii)	$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \int -\sqrt{\frac{k}{m}} dt$ $\arcsin\left(\frac{x}{a}\right) = -\sqrt{\frac{k}{m}} t + c_2$ $x = a, t = 0 \Rightarrow \frac{1}{2}\pi = c_2$ $x = a \sin\left(\frac{1}{2}\pi - \sqrt{\frac{k}{m}} t\right) = a \cos\left(\sqrt{\frac{k}{m}} t\right)$	M1* A1 DM1 A1 [4]	Solutions from SHM NOT acceptable  Accept $c_2 = \arcsin 1$  Either form
3	(i)	$l = 2a \sin \theta$ $V = \frac{\lambda}{2a} (2a \sin \theta - a)^2$ $\dots + mga \cos 2\theta$ $\frac{dV}{d\theta} = -2mga \sin 2\theta + \frac{\lambda}{a} (2a \sin \theta - a) \cdot 2a \cos \theta$ $= -4mga \sin \theta \cos \theta + 2\lambda a \cos \theta (2 \sin \theta - 1)$ $= 2a \cos \theta (2\lambda \sin \theta - 2mg \sin \theta - \lambda)$	M1 A1 M1 A1 M1 M1 E1 [7]	OE eg $a\sqrt{2 - 2\cos 2\theta}$ EPE OE eg $\frac{\lambda}{2a} (a\sqrt{2 - 2\cos 2\theta} - a)^2$ Both terms GPE OE eg $mga \sin(\frac{1}{2}\pi - 2\theta)$  Differentiate  Use trigonometric identities as necessary

Question		Answer	Marks	Guidance
3	(ii)	$\theta = \frac{1}{2}\pi \Rightarrow \frac{dV}{d\theta} = 0 \times (\dots) = 0$ hence equilibrium $\frac{d^2V}{d\theta^2} = -2a \sin \theta (2\lambda \sin \theta - 2mg \sin \theta - \lambda) + 2a \cos \theta (2\lambda \cos \theta - 2mg \cos \theta)$ $\theta = \frac{1}{2}\pi \Rightarrow \frac{d^2V}{d\theta^2} = -2a(2\lambda - 2mg - \lambda)$ So $\lambda < 2mg \Rightarrow \frac{d^2V}{d\theta^2} > 0 \Rightarrow$ stable If $\cos \theta \neq 0$ $\frac{dV}{d\theta} = 0 \Leftrightarrow 2\lambda \sin \theta - 2mg \sin \theta - \lambda = 0$ $\Leftrightarrow \sin \theta = \frac{\lambda}{2\lambda - 2mg}$ But $\lambda < 2mg \Rightarrow 2\lambda - 2mg < \lambda$ $\Rightarrow \sin \theta > 1$ or $\sin \theta < 0$ So no valid solutions	M1  E1  M1 A1  M1  E1  M1  M1 E1 [9]	Here or in (iii) or use sign method  Use $V''$ or equivalent method  Consider other solutions  Attempt at showing not valid Must consider both ends
3	(iii)	If $\lambda > 2mg, \theta = \frac{1}{2}\pi$ as before $V'' < 0$ so unstable or $\sin \theta = \frac{\lambda}{2\lambda - 2mg}$ and $\frac{1}{2} < \frac{\lambda}{2\lambda - 2mg} < 1$ so gives valid solution $\theta = \sin^{-1}\left(\frac{\lambda}{2\lambda - 2mg}\right)$ or $\pi - \sin^{-1}\left(\frac{\lambda}{2\lambda - 2mg}\right)$ and $V'' = 0 + 2a \cos^2 \theta (2\lambda - 2mg)$ $= (+ve)(+ve)$ so stable (in both cases)	B1  B1  E1  E1  B1  M1 A1 [7]	For both

Question		Answer	Marks	Guidance
4	(i)	$\delta I = 2\pi r \delta r \rho r^2$ $\rho = \frac{m}{\pi a^2}$ $I_{\text{disc}} = \int_0^a \frac{m}{2a^2} r^3 dr$ $= \frac{m}{2a^2} \left[ \frac{1}{4} r^4 \right]_0^a$ $= \frac{1}{2} m a^2$	B1 B1 M1 M1 A1 E1 [6]	for $k \int r^3 dr$ $k \left[ \frac{1}{4} r^4 \right]_0^a$ with limits $\frac{k}{4} a^4$
4	(ii)	$I = m_1 a^2 + \frac{1}{2} m a^2 \times 2$ $m = M \frac{\pi a^2}{2\pi a^2 + 2\pi ah}$ $m_1 = M \frac{2\pi ah}{2\pi a^2 + 2\pi ah}$ $\text{So } I = Ma^2 \left( \frac{\pi a^2 + 2\pi ah}{2\pi a^2 + 2\pi ah} \right)$ $I = \frac{1}{2} Ma^2 \left( \frac{a + 2h}{a + h} \right)$	M1 M1 B1 B1 M1 E1 [6]	Curved surface $2\pi h \rho a^3$ Combine $+ \frac{1}{2} \rho \pi a^4 \times 2$ $m = \pi \rho a^2$ $m_1 = 2\pi ah \rho$ Substitute $I = M \frac{\pi \rho a^4 + 2\pi \rho a^3}{2\pi \rho a^2 + 2\pi \rho ah}$
4	(iii)	$I = \frac{1}{2} \times 8 \times 0.5^2 \left( \frac{0.5 + 0.6}{0.5 + 0.3} \right) = 1.375$ $I(\omega - 0) = Ja$ $1.375\omega = 55 \times 0.5$ $\omega = 20 \text{ rad s}^{-1}$	B1 M1 A1 [3]	Impulse/moment

Question		Answer	Marks	Guidance
4	(iv)	$I \frac{d\dot{\theta}}{dt} = -2\dot{\theta}^2$ $\int 1.375\dot{\theta}^{-2} d\dot{\theta} = \int -2 dt$ $-\frac{1.375}{\dot{\theta}} = -2t + c$ $t=0, \dot{\theta}=20 \Rightarrow c=-0.06875$ $t=5 \Rightarrow -\frac{1.375}{\dot{\theta}} = -10 - 0.06875$ $\Rightarrow \dot{\theta} = 0.137 \text{ (3sf)}$	B1 M1 M1 A1 M1 M1 A1 [7]	Separate Integrate Use condition
4	(v)	$I \left( \frac{-0.137}{t} \right) = -0.03$ $t = 6.26 \text{ s}$	M1 A1 A1 [3]	Complete method with correct acceleration (or both sides +ve) awfw [6.25, 6.3] CAO

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# 4764 Mechanics 4

## General Comments

The performance of the candidates was generally very good, as is usual on this paper. Most candidates showed good knowledge of the techniques and concepts examined. The standard of presentation was very high, though many candidates did not give sufficient intermediate steps when working towards a given answer.

## Comments on Individual Questions

- 1) (Variable mass – Rocket in deep space)
  - (i) Almost all candidates knew to consider momentum-impulse and knew the basic technique. However, the detail was not often well-understood; many candidates took  $\delta m > 0$  and worked with the rocket's mass changing from  $m$  to  $m - \delta m$  rather than either using  $\delta m < 0$  and the mass changing to  $m + \delta m$  or using  $|\delta m|$ .
  - (ii) This was very well answered in general, with only a few candidates making errors in their integration or subsequent manipulation.
- 2) (Variable force)
  - (i) This was not well answered by the majority of candidates; many ignored the presence of gravity and/or neglected to take into account the tension in the string at the equilibrium point.
  - (ii) The solution required a relatively simple separation of variables and was well answered by the majority of candidates, including reasonable descriptions of why the negative root is chosen.  
Some candidates approached this from SHM or by energy considerations, which was acceptable in this part.
  - (iii) Most candidates understood which form of Newton's second law was required here. Those that recognised, or looked up, the relevant integral to get arcsine tended to get full marks.  
Since the question asked specifically for candidates to integrate, marks were not given for solutions based on energy or SHM considerations.
- 3) (Equilibrium)
  - (i) This type of question was obviously familiar to most candidates and they performed the necessary trigonometry and calculus with accuracy.
  - (ii) The idea that  $dV/d\theta = 0$  at a point of equilibrium was well understood, as was the process of deciding on stability by using the second derivative of  $V$ . Most candidates found it difficult to explain why there were no other points of equilibrium and most failed to cover all the possibilities.
  - (iii) Most candidates stated that  $\theta = \pi/2$  was still a point of equilibrium, but many candidates did not realise that they needed to reconsider whether or not it was still stable.  
Dealing with the other two points was found to be difficult by most, in particular very few candidates checked that the value of  $\sin\theta$  was valid, and most did not find the second solution in the domain. The manipulation to show the stability of these two solutions was quite tricky, but some candidates were able to do so with great skill.

4) *(Rotation)*

- (i) The proof of this standard result about discs was well done by the majority of candidates. Some chose to take the mass per unit area to be 1, but only those that did so explicitly were awarded full marks.
- (ii) Many candidates found this question very difficult. In general, the expressions for the mass or moment of inertia of the ends and the curved surface were found, but the necessary manipulation to eliminate the mass per unit area, or to deal with the masses as proportions of the whole mass, was not well done.
- (iii) This was well answered by the majority of candidates. Some neglected the factor of 0.5 for the impulse, but all incorrect values were followed through into the next part.
- (iv) Many candidates had the sign of the couple as positive rather than negative here, which meant that they had the wrong differential equation to start with. However, those that could get started on the integrals could generally make their way through to a correct answer.
- (v) Those candidates that attempted this part generally answered it very well, but many did not get this far through Q4.

<b>GCE Mathematics (MEI)</b>											
			<b>Max Mark</b>	<b>90% cp</b>	<b>a</b>	<b>b</b>	<b>c</b>	<b>d</b>	<b>e</b>	<b>u</b>	
4753/01	(C3) MEI Methods for Advanced Mathematics with Coursework: Written Paper		Raw 100	72 18 18 100	66 16 16 90	60 15 15 80	53 13 13 70	47 11 11 60	41 9 9 50	34 8 8 40	0 0 0 0
4753/02	(C3) MEI Methods for Advanced Mathematics with Coursework: Coursework		Raw UMS	90 100	73 90	65 80	57 70	50 60	43 50	36 40	0 0
4753/82	(C3) MEI Methods for Advanced Mathematics with Coursework: Carried Forward Coursework Mark		Raw	18	16	15	13	11	9	8	0
4753	(C3) MEI Methods for Advanced Mathematics with Coursework		UMS	100	90	80	70	60	50	40	0
4754/01	(C4) MEI Applications of Advanced Mathematics		Raw UMS	90 100	73 90	65 80	57 70	50 60	43 50	36 40	0 0
4756/01	(FP2) MEI Further Methods for Advanced Mathematics		Raw UMS	72 100	66 90	61 80	53 70	46 60	39 50	32 40	0 0
4757/01	(FP3) MEI Further Applications of Advanced Mathematics		Raw UMS	72 100	61 90	54 80	47 70	40 60	34 50	28 40	0 0

4758/01 (DE) MEI Differential Equations with Coursework: Written Paper	Raw 18	72	68	63	57	51	45	39	0
4758/02 (DE) MEI Differential Equations with Coursework: Coursework	Raw 18	16	15	13	11	9	8	0	0
4758/82 (DE) MEI Differential Equations with Coursework: Carried Forward Coursework Mark	Raw 100	18	16	15	13	11	9	8	0
4758 (DE) MEI Differential Equations with Coursework	UMS 100	90	80	70	60	50	40	0	0
4762/01 (M2) MEI Mechanics 2	Raw 100	72	65	58	51	44	38	32	0
	UMS 100	90	80	70	60	50	40	0	0
4763/01 (M3) MEI Mechanics 3	Raw 100	72	67	63	56	50	44	38	0
	UMS 100	90	80	70	60	50	40	0	0
4764/01 (M4) MEI Mechanics 4	Raw 100	72	63	56	49	42	35	29	0
	UMS 100	90	80	70	60	50	40	0	0
4767/01 (S2) MEI Statistics 2	Raw 100	72	66	61	55	49	43	38	0
	UMS 100	90	80	70	60	50	40	0	0
4768/01 (S3) MEI Statistics 3	Raw 100	72	65	58	51	44	38	32	0
	UMS 100	90	80	70	60	50	40	0	0
4769/01 (S4) MEI Statistics 4	Raw 100	72	63	56	49	42	35	28	0
	UMS 100	90	80	70	60	50	40	0	0
4772/01 (D2) MEI Decision Mathematics 2	Raw 100	72	62	56	50	44	39	34	0
	UMS 100	90	80	70	60	50	40	0	0
4773/01 (DC) MEI Decision Mathematics Computation	Raw 100	72	52	46	40	34	29	24	0
	UMS 100	90	80	70	60	50	40	0	0
4777/01 (NC) MEI Numerical Computation	Raw 100	72	63	55	47	39	32	25	0
	UMS 100	90	80	70	60	50	40	0	0