OXFORD CAMBRIDGE AND RSA EXAMINATIONS

AS GCE

4755

MATHEMATICS (MEI)

Further Concepts for Advanced Mathematics (FP1)

QUESTION PAPER

FRIDAY 18 MAY 2012: Morning

DURATION: 1 hour 30 minutes
plus your additional time allowance

MODIFIED ENLARGED

Candidates answer on the Printed Answer Book or any suitable paper provided by the centre. The Printed Answer Book may be enlarged by the centre.

OCR SUPPLIED MATERIALS:

Printed Answer Book 4755
MEI Examination Formulae and Tables (MF2)

OTHER MATERIALS REQUIRED:

Scientific or graphical calculator

READ INSTRUCTIONS OVERLEAF

DC (SLM) 12MEP06301 63635/1

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- **The Question Paper will be found in the centre of the Printed Answer Book.**
- **Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.**
- **WRITE YOUR ANSWER TO EACH QUESTION IN THE SPACE PROVIDED IN THE PRINTED ANSWER BOOK. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).**
- **Use black ink. HB pencil may be used for graphs and diagrams only.**
- **Read each question carefully. Make sure you know what you have to do before starting your answer.**
- **Answer ALL the questions.**
- **You are permitted to use a scientific or graphical calculator in this paper.**
- **Final answers should be given to a degree of accuracy appropriate to the context.**

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- **The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.**
- **You are advised that an answer may receive NO MARKS unless you show sufficient detail of the working to indicate that a correct method is being used.**
- **The total number of marks for this paper is 72.**

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- **Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.**

SECTION A (36 marks)

1 You are given that the matrix $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ represents a transformation A, and that the matrix $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ represents a transformation B.

(i) Describe the transformations A and B. [2]

(ii) Find the matrix representing the composite transformation consisting of A followed by B. [2]

(iii) What single transformation is represented by this matrix? [1]

2 You are given that z_1 and z_2 are complex numbers.

$z_1 = 3 + 3\sqrt{3}\mathrm{j}$, and z_2 has modulus 5 and argument $\frac{\pi}{3}$.

(i) Find the modulus and argument of z_1, giving your answers exactly. [4]

(ii) Express z_2 in the form $a + b\mathrm{j}$, where a and b are to be given exactly. [2]

(iii) Explain why, when plotted on an Argand diagram, z_1, z_2 and the origin lie on a straight line. [1]

3 The cubic equation $3x^3 + 8x^2 + px + q = 0$ has roots α, $\frac{\alpha}{6}$ and $\alpha - 7$. Find the values of α, p and q. [6]

4 Solve the inequality $\frac{3}{x-4} > 1$. [4]

5 (i) Show that $\frac{1}{2r+1} - \frac{1}{2r+3} \equiv \frac{2}{(2r+1)(2r+3)}$. [2]

(ii) Use the method of differences to find $\sum_{r=1}^{30} \frac{1}{(2r+1)(2r+3)}$, expressing your answer as a fraction. [5]

6 A sequence is defined by $a_1 = 1$ and $a_{k+1} = 3(a_k + 1)$.

(i) Calculate the value of the third term, a_3. [1]

(ii) Prove by induction that $a_n = \frac{5 \times 3^{n-1} - 3}{2}$. [6]

SECTION B (36 marks)

7 **A curve has equation $y = \dfrac{x^2 - 25}{(x - 3)(x + 4)(3x + 2)}$.**

(i) **Write down the coordinates of the points where the curve crosses the axes. [3]**

(ii) **Write down the equations of the asymptotes. [4]**

(iii) **Determine how the curve approaches the horizontal asymptote for large positive values of x, and for large negative values of x. [3]**

(iv) **Sketch the curve. [4]**

8 **(i)** **Verify that $1 + 3j$ is a root of the equation $3z^3 - 2z^2 + 22z + 40 = 0$, showing your working. [4]**

(ii) **Explain why the equation must have exactly one real root. [1]**

(iii) **Find the other roots of the equation. [5]**

9 You are given that $\mathbf{A} = \begin{pmatrix} -3 & -4 & 1 \\ 2 & 1 & k \\ 7 & -1 & -1 \end{pmatrix}$,

$\mathbf{B} = \begin{pmatrix} -4 & -5 & 11 \\ -19 & -4 & -7 \\ -9 & -31 & 2-k \end{pmatrix}$ and

$\mathbf{AB} = \begin{pmatrix} 79 & 0 & -3-k \\ -9k-27 & -31k-14 & q \\ p & 0 & 82+k \end{pmatrix}$ where

p and q are to be determined.

(i) Show that $p = 0$ and $q = 15 + 2k - k^2$. [3]

It is now given that $k = -3$.

(ii) Find AB and hence write down the inverse matrix $\mathbf{A}^{-1}$. [5]

(iii) Use a matrix method to find the values of x, y and z that satisfy the equation $\mathbf{A}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 14 \\ -23 \\ 9 \end{pmatrix}$. [4]