OCR
RECOGNISING ACHIEVEMENT

Friday 18 May 2012 – Morning

AS GCE MATHEMATICS (MEI)

4755 Further Concepts for Advanced Mathematics (FP1)

QUESTION PAPER

4715720612

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4755
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.
- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.
- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (36 marks)

1 You are given that the matrix $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ represents a transformation A, and that the matrix $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ represents a transformation B.

(i) Describe the transformations A and B. **[2]**

(ii) Find the matrix representing the composite transformation consisting of A followed by B. **[2]**

(iii) What single transformation is represented by this matrix? **[1]**

2 You are given that z_1 and z_2 are complex numbers.

$z_1 = 3 + 3\sqrt{3}\text{j}$, and z_2 has modulus 5 and argument $\frac{\pi}{3}$.

(i) Find the modulus and argument of z_1, giving your answers exactly. **[4]**

(ii) Express z_2 in the form $a + b\text{j}$, where a and b are to be given exactly. **[2]**

(iii) Explain why, when plotted on an Argand diagram, z_1, z_2 and the origin lie on a straight line. **[1]**

3 The cubic equation $3x^3 + 8x^2 + px + q = 0$ has roots α, $\frac{\alpha}{6}$ and $\alpha - 7$. Find the values of α, p and q. **[6]**

4 Solve the inequality $\frac{3}{x-4} > 1$. **[4]**

5 **(i)** Show that $\frac{1}{2r+1} - \frac{1}{2r+3} \equiv \frac{2}{(2r+1)(2r+3)}$. **[2]**

(ii) Use the method of differences to find $\sum_{r=1}^{30} \frac{1}{(2r+1)(2r+3)}$, expressing your answer as a fraction. **[5]**

6 A sequence is defined by $a_1 = 1$ and $a_{k+1} = 3(a_k + 1)$.

(i) Calculate the value of the third term, a_3. **[1]**

(ii) Prove by induction that $a_n = \frac{5 \times 3^{n-1} - 3}{2}$. **[6]**

Section B (36 marks)

7 A curve has equation $y = \dfrac{x^2 - 25}{(x-3)(x+4)(3x+2)}$.

(i) Write down the coordinates of the points where the curve crosses the axes. **[3]**

(ii) Write down the equations of the asymptotes. **[4]**

(iii) Determine how the curve approaches the horizontal asymptote for large positive values of x, and for large negative values of x. **[3]**

(iv) Sketch the curve. **[4]**

8 **(i)** Verify that 1 + 3j is a root of the equation $3z^3 - 2z^2 + 22z + 40 = 0$, showing your working. **[4]**

(ii) Explain why the equation must have exactly one real root. **[1]**

(iii) Find the other roots of the equation. **[5]**

9 You are given that $\mathbf{A} = \begin{pmatrix} -3 & -4 & 1 \\ 2 & 1 & k \\ 7 & -1 & -1 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} -4 & -5 & 11 \\ -19 & -4 & -7 \\ -9 & -31 & 2-k \end{pmatrix}$ and

$\mathbf{AB} = \begin{pmatrix} 79 & 0 & -3-k \\ -9k-27 & -31k-14 & q \\ p & 0 & 82+k \end{pmatrix}$ where p and q are to be determined.

(i) Show that $p = 0$ and $q = 15 + 2k - k^2$. **[3]**

It is now given that $k = -3$.

(ii) Find **AB** and hence write down the inverse matrix $\mathbf{A}^{-1}$. **[5]**

(iii) Use a matrix method to find the values of x, y and z that satisfy the equation $\mathbf{A}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 14 \\ -23 \\ 9 \end{pmatrix}$. **[4]**

THERE ARE NO QUESTIONS WRITTEN ON THIS PAGE