

**Friday 18 May 2012 – Morning**

**AS GCE MATHEMATICS (MEI)**

**4755** Further Concepts for Advanced Mathematics (FP1)

**QUESTION PAPER**

Candidates answer on the Printed Answer Book.

**OCR supplied materials:**

- Printed Answer Book 4755
- MEI Examination Formulae and Tables (MF2)

**Other materials required:**

- Scientific or graphical calculator

**Duration:** 1 hour 30 minutes



**INSTRUCTIONS TO CANDIDATES**

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

**INFORMATION FOR CANDIDATES**

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

**INSTRUCTION TO EXAMS OFFICER/INVIGILATOR**

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

## Section A (36 marks)

- 1 You are given that the matrix  $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$  represents a transformation A, and that the matrix  $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$  represents a transformation B.
- (i) Describe the transformations A and B. [2]
- (ii) Find the matrix representing the composite transformation consisting of A followed by B. [2]
- (iii) What single transformation is represented by this matrix? [1]
- 2 You are given that  $z_1$  and  $z_2$  are complex numbers.  
 $z_1 = 3 + 3\sqrt{3}j$ , and  $z_2$  has modulus 5 and argument  $\frac{\pi}{3}$ .
- (i) Find the modulus and argument of  $z_1$ , giving your answers exactly. [4]
- (ii) Express  $z_2$  in the form  $a + bj$ , where  $a$  and  $b$  are to be given exactly. [2]
- (iii) Explain why, when plotted on an Argand diagram,  $z_1$ ,  $z_2$  and the origin lie on a straight line. [1]
- 3 The cubic equation  $3x^3 + 8x^2 + px + q = 0$  has roots  $\alpha$ ,  $\frac{\alpha}{6}$  and  $\alpha - 7$ . Find the values of  $\alpha$ ,  $p$  and  $q$ . [6]
- 4 Solve the inequality  $\frac{3}{x-4} > 1$ . [4]
- 5 (i) Show that  $\frac{1}{2r+1} - \frac{1}{2r+3} \equiv \frac{2}{(2r+1)(2r+3)}$ . [2]
- (ii) Use the method of differences to find  $\sum_{r=1}^{30} \frac{1}{(2r+1)(2r+3)}$ , expressing your answer as a fraction. [5]
- 6 A sequence is defined by  $a_1 = 1$  and  $a_{k+1} = 3(a_k + 1)$ .
- (i) Calculate the value of the third term,  $a_3$ . [1]
- (ii) Prove by induction that  $a_n = \frac{5 \times 3^{n-1} - 3}{2}$ . [6]

## Section B (36 marks)

- 7 A curve has equation  $y = \frac{x^2 - 25}{(x - 3)(x + 4)(3x + 2)}$ .
- (i) Write down the coordinates of the points where the curve crosses the axes. [3]
- (ii) Write down the equations of the asymptotes. [4]
- (iii) Determine how the curve approaches the horizontal asymptote for large positive values of  $x$ , and for large negative values of  $x$ . [3]
- (iv) Sketch the curve. [4]
- 8 (i) Verify that  $1 + 3j$  is a root of the equation  $3z^3 - 2z^2 + 22z + 40 = 0$ , showing your working. [4]
- (ii) Explain why the equation must have exactly one real root. [1]
- (iii) Find the other roots of the equation. [5]

- 9 You are given that  $\mathbf{A} = \begin{pmatrix} -3 & -4 & 1 \\ 2 & 1 & k \\ 7 & -1 & -1 \end{pmatrix}$ ,  $\mathbf{B} = \begin{pmatrix} -4 & -5 & 11 \\ -19 & -4 & -7 \\ -9 & -31 & 2 - k \end{pmatrix}$  and

$$\mathbf{AB} = \begin{pmatrix} 79 & 0 & -3 - k \\ -9k - 27 & -31k - 14 & q \\ p & 0 & 82 + k \end{pmatrix} \text{ where } p \text{ and } q \text{ are to be determined.}$$

- (i) Show that  $p = 0$  and  $q = 15 + 2k - k^2$ . [3]

It is now given that  $k = -3$ .

- (ii) Find  $\mathbf{AB}$  and hence write down the inverse matrix  $\mathbf{A}^{-1}$ . [5]
- (iii) Use a matrix method to find the values of  $x$ ,  $y$  and  $z$  that satisfy the equation  $\mathbf{A} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 14 \\ -23 \\ 9 \end{pmatrix}$ . [4]

**THERE ARE NO QUESTIONS WRITTEN ON THIS PAGE**



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