OXFORD CAMBRIDGE AND RSA EXAMINATIONS

AS GCE

4761

MATHEMATICS (MEI)

Mechanics 1

QUESTION PAPER

WEDNESDAY 25 JANUARY 2012: Afternoon
DURATION: 1 hour 30 minutes

SUITABLE FOR VISUALLY IMPAIRED CANDIDATES

Candidates answer on the Printed Answer Book or any suitable paper provided by the centre. The Printed Answer Book may be enlarged by the centre.

OCR SUPPLIED MATERIALS:

Printed Answer Book 4761
Insert for Question 5 (inserted)
MEI Examination Formulae and Tables (MF2)

OTHER MATERIALS REQUIRED:

Scientific or graphical calculator

READ INSTRUCTIONS OVERLEAF

DC (KN/SW) 12MEP01020 55373

INSTRUCTIONS TO CANDIDATES

- **Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.**
- **WRITE YOUR ANSWER TO EACH QUESTION IN THE SPACE PROVIDED IN THE PRINTED ANSWER BOOK. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).**
- **Use black ink. HB pencil may be used for graphs and diagrams only.**
- **Answer ALL the questions.**
- **Read each question carefully. Make sure you know what you have to do before starting your answer.**
- **You are permitted to use a scientific or graphical calculator in this paper.**
- **Final answers should be given to a degree of accuracy appropriate to the context.**
- **The acceleration due to gravity is denoted by $g\,\text{m}\,\text{s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.**

INFORMATION FOR CANDIDATES

- **The number of marks is given in brackets [ ] at the end of each question or part question on the Question Paper.**
- **You are advised that an answer may receive NO MARKS unless you show sufficient detail of the working to indicate that a correct method is being used.**
- **The total number of marks for this paper is 72.**

SECTION A (36 marks)

1 Fig. 1 below shows two blocks of masses 3 kg and 5 kg connected by a light string which passes over a smooth, fixed pulley.

Initially the blocks are held at rest but then they are released.

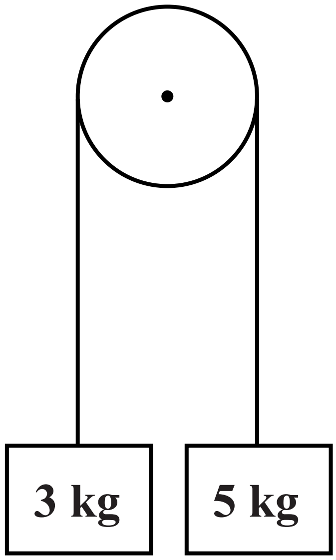

Fig. 1

Find the acceleration of the blocks when they start to move, and the tension in the string. [5]

2 **Fig. 2 below shows a small object, P, of weight 20 N, suspended by two light strings. The strings are tied to points A and B on a sloping ceiling which is at an angle of 60° to the upward vertical. The string AP is at 60° to the downward vertical and the string BP makes an angle of 30° with the ceiling.**

The object is in equilibrium.

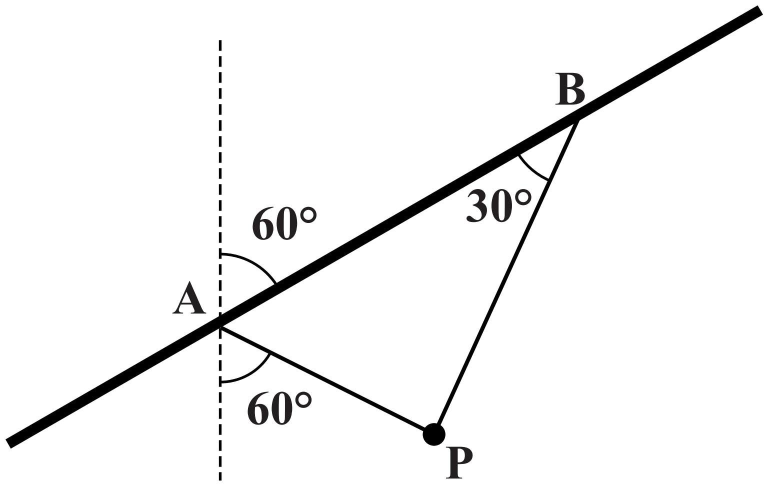

Fig. 2

(i) **Show that ∠APB = 90°. [1]**

(ii) **Draw a labelled triangle of forces to represent the three forces acting on P. [3]**

(iii) **Hence, or otherwise, find the tensions in the two strings. [3]**

3 Two girls, Marie and Nina, are members of an Olympic hockey team. They are doing fitness training.

Marie runs along a straight line at a constant speed of $6\,\text{m}\,\text{s}^{-1}$.

Nina is stationary at a point O on the line until Marie passes her. Nina immediately runs after Marie until she catches up with her.

The time, t s, is measured from the moment when Nina starts running. So when $t = 0$, both girls are at O.

Nina's acceleration, a $\text{m}\,\text{s}^{-2}$, is given by

$a = 4 - t$ **for** $0 \leqslant t \leqslant 4$,

$a = 0$ **for** $t > 4$.

(i) Show that Nina's speed, $v\,\text{m}\,\text{s}^{-1}$, is given by

$v = 4t - \frac{1}{2}t^2$ **for** $0 \leqslant t \leqslant 4$,

$v = 8$ **for** $t > 4$. **[3]**

(ii) Find an expression for the distance Nina has run at time t, for $0 \leqslant t \leqslant 4$.

Find how far Nina has run when $t = 4$ and when $t = 5\frac{1}{3}$. [4]

(iii) Show that Nina catches up with Marie when $t = 5\frac{1}{3}$. [1]

4 **A projectile P travels in a vertical plane over level ground. Its position vector r at time t seconds after projection is modelled by**

$$\mathbf{r} = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 5 \end{pmatrix} + \begin{pmatrix} 30 \\ 40 \end{pmatrix} t - \begin{pmatrix} 0 \\ 5 \end{pmatrix} t^2 ,$$

where distances are in metres and the origin is a point on the level ground.

(i) Write down

(*A*) the height from which P is projected,

(*B*) the value of g in this model. [2]

(ii) Find the displacement of P from $t = 3$ to $t = 5$. [2]

(iii) Show that the equation of the trajectory is

$$y = 5 + \frac{4}{3}x - \frac{x^2}{180} . \quad [4]$$

5 The vectors p and q are given by $\mathbf{p} = 8\mathbf{i} + \mathbf{j}$ and $\mathbf{q} = 4\mathbf{i} - 7\mathbf{j}$.

(i) Show that p and q are equal in magnitude. [3]

(ii) Show that p + q is parallel to $2\mathbf{i} - \mathbf{j}$. [2]

(iii) Draw p + q and p − q on the grid. [You may use the insert provided.]

Write down the angle between these two vectors. [3]

SECTION B (36 marks)

6 Robin is driving a car of mass 800 kg along a straight horizontal road at a speed of $40\,\mathrm{m\,s^{-1}}$.

Robin applies the brakes and the car decelerates uniformly; it comes to rest after travelling a distance of 125 m.

(i) Show that the resistance force on the car when the brakes are applied is 5120 N. [4]

(ii) Find the time the car takes to come to rest. [2]

For the rest of this question, assume that when Robin applies the brakes there is a constant resistance force of 5120 N on the car.

The car returns to its speed of $40\,\mathrm{m\,s^{-1}}$ and the road remains straight and horizontal.

Robin sees a red light 155 m ahead, takes a short time to react and then applies the brakes.

The car comes to rest before it reaches the red light.

(iii) Show that Robin's reaction time is less than 0.75 s. [2]

The 'stopping distance' is the total distance travelled while a driver reacts and then applies the brakes to bring the car to rest. For the rest of this question, assume that Robin is still driving the car described above and has a reaction time of 0.675 s. (This is the figure used in calculating the stopping distances given in the Highway Code.)

(iv) Calculate the stopping distance when Robin is driving at 20 ms^{-1} on a horizontal road. [3]

The car then travels down a hill which has a slope of 5° to the horizontal.

(v) Find the stopping distance when Robin is driving at 20 ms^{-1} down this hill. [6]

(vi) By what percentage is the stopping distance increased by the fact that the car is going down the hill? Give your answer to the nearest 1%. [1]

7 **Fig. 7 below shows the trajectory of an object which is projected from a point O on horizontal ground. Its initial velocity is 40 ms^{-1} at an angle of α to the horizontal.**

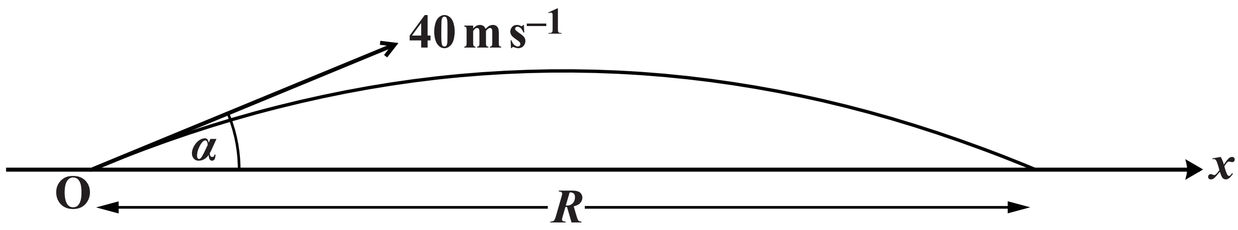

Fig. 7

(i) **Show that, according to the standard projectile model in which air resistance is neglected, the flight time, T s, and the range, R m, are given by**

$$T = \frac{80 \sin \alpha}{g}$$

and

$$R = \frac{3200 \sin \alpha \cos \alpha}{g} .$$ **[6]**

A company is designing a new type of ball and wants to model its flight.

(ii) **Initially the company uses the standard projectile model.**

Use this model to show that when $\alpha = 30°$ and the initial speed is 40 ms^{-1}, T is approximately 4.08 and R is approximately 141.4.

Find the values of T and R when $\alpha = 45°$. [3]

The company tests the ball using a machine that projects it from ground level across horizontal ground. The speed of projection is set at 40 ms^{-1}.

When the angle of projection is set at 30°, the range is found to be 125 m.

(iii) Comment briefly on the accuracy of the standard projectile model in this situation. [1]

The company refines the model by assuming that the ball has a constant deceleration of 2 ms^{-2} in the horizontal direction.

In this new model, the resistance to the vertical motion is still neglected and so the flight time is still 4.08 s when the angle of projection is 30°.

(iv) Using the new model, with $\alpha = 30°$, show that the horizontal displacement from the point of projection, x m at time t s, is given by $x = 40t \cos 30° - t^2$.

Find the range and hence show that this new model is reasonably accurate in this case. [4]

The company then sets the angle of projection to 45° while retaining a projection speed of 40 $m\,s^{-1}$. With this setting the range of the ball is found to be 135 m.

(v) Investigate whether the new model is also accurate for this angle of projection. [3]

(vi) Make one suggestion as to how the model could be further refined. [1]