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RECOGNISING ACHIEVEMENT

GCE

Mathematics (MEI)

Advanced GCE

Unit **4768:** Statistics 3

Mark Scheme for January 2012

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All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

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Annotations and abbreviations

Annotation in scoris	**Meaning**
✓and ✖	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics (MEI) Statistics strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E

A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (e.g. 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance
1	(a)		Simple random sampling is when every sample of the required size … has an equal chance of being chosen. eg One needs access to the entire population in order to establish the sampling frame.	 E1 E1 E2 **[4]**	SC Allow E1 for "Every member of the population has an equal chance of being chosen". E2, 1, 0. Reward any sensible point. SC1 "Sample may not be representative of the population" oe.
1	(b)	(i)	$H_0 : \mu = 70.3$ $H_1 : \mu < 70.3$ Where μ is the (population) mean drying rate. $\bar{x} = 67.675$, $s_{n-1} = 4.129$ ($s_{n-1}^2 = 17.049$) Test statistic is $\dfrac{67.675 - 70.3}{\left(\dfrac{4.129}{\sqrt{12}}\right)}$ $= -2.202(2)$ Refer to t_{11}. Lower 5% point is –1·796. –2.202 < –1·796, ∴ Result is significant. Seems mean drying rate has reduced. Underlying population is Normal.	B1 B1 B1 M1 A1 M1 A1 A1 A1 B1 **[10]**	Both. Hypotheses in words only must include "population". Do NOT allow "$\bar{X} = ...$" or similar unless $\bar{X}$ is clearly and explicitly stated to be a population mean. For adequate verbal definition. Allow absence of "population" if correct notation μ is used. Do not allow $s_n = 3.953$ ($s_n^2 = 15.629$). Allow c's $\bar{x}$ and/or s_{n-1}. Allow alternative: 70.3 + (c's –1·796) × $\frac{4.129}{\sqrt{12}}$ (= 68.15(9)) for subsequent comparison with $\bar{x}$. (Or $\bar{x}$ – (c's –1·796) × $\frac{4.129}{\sqrt{12}}$ (= 69.81(6)) for comparison with 70.3) cao but ft from here in any case if wrong. Use of 70.3 – $\bar{x}$ scores M1A0, but ft. No ft from here if wrong. Allow any t_{11} value from tables. c.a.o. No ft from here if wrong. Must be –1·796 unless it is clear that absolute values are being used. P(t < –2.202) = 0.0249. ft only c's test statistic. ft only c's test statistic. "Non-assertive" conclusion in context to include "on average" oe.

Question			Answer	Marks	Guidance
1	(b)	(ii)	CI is given by $67.675 \pm 2{\cdot}201 \times \frac{4.129}{\sqrt{12}}$ $= 67.675 \pm 2.623 = (65.05(2), 70.29(8)$	M1 B1 M1 A1 **[4]**	ZERO if not same distribution as test. Same wrong distribution scores max M1B0M1A0. Recovery to t_{11} is OK. ft c's $\bar{x} \pm$. ft c's s_{n-1}. cao Must be expressed as an interval. (t= 1.796 gives (65.534, 69.816) (M1B0M1A0)).
2			$S \sim \text{N}(505,\ 11^2)$ $L \sim \text{N}(1005,\ 17^2)$		When a candidate's answers suggest that (s)he appears to have neglected to use the difference columns of the Normal distribution tables, penalise the first occurrence only.
	(i)		$\text{P}(995 < L < 1020)$ $= \text{P}\left(\frac{995-1005}{17} < Z < \frac{1020-1005}{17}\right)$ $= \text{P}(-0.5882 < Z < 0.8824)$ $= 0.8113 - (1 - 0.7218)$ $= 0.5331$	 M1 A1 A1 **[3]**	 For standardising. Award once, here or elsewhere. cao
2	(ii)		$S_1 - S_2 \sim \text{N}(0,\ 11^2 + 11^2 = 242)$ $\text{P}(-25 < S_1 - S_2 < 25)$ $= \text{P}\left(\frac{-25-0}{\sqrt{242}} < Z < \frac{25-0}{\sqrt{242}}\right)$ $= \text{P}(-1.607 < Z < 1.607)$ $= 2 \times (0.9459 - 0.5)$ $= 0.8918$	B1 M1 A1 **[3]**	Mean and variance. Accept sd = $\sqrt{242}$ = 15.55... Formulate the problem. cao

Question			Answer	Marks	Guidance
2	(iii)		Want P($S_1 + S_2 > L$) i.e. P($S_1 + S_2 - L > 0$) $S_1 + S_2 - L \sim$ N(505 + 505 – 1005 = 5, $11^2 + 11^2 + 17^2 = 531$) P(this > 0) = P($Z > \frac{0-5}{\sqrt{531}} = -0.2170$) = 0·5859	M1 B1 B1 A1 **[4]**	Allow $L - (S_1 + S_2)$ provided subsequent work is consistent. Mean Variance. Accept sd = √531 = 23.04... cao
2	(iv)		Want P($S > ½L + 5$) i.e. P($S - ½L > 5$) $S - ½L \sim$ N(505 – 1005/2 = 2.5, $11^2 + 17^2/2^2 = 193.25$) P(this > 5) = P($Z > \frac{5-2.5}{\sqrt{193.25}} = 0.1798$) = 1 – 0.5714 = 0.4286	M1 B1 B1 A1 **[4]**	Allow $½L - S$ provided subsequent work is consistent. Mean. Variance. Accept sd = √193.25 = 13.90... cao
2	(v)		CI is given by 246 ± 1.96 $\times \frac{14}{\sqrt{20}}$ = 246 ± 6.1358= (239.8(6), 252.1(3))	 M1 B1 M1 A1 **[4]**	 Must be 1.96. Anything else can get M1B0M1A0 max. cao Must be expressed as an interval.
3	(a)		H_0: The model for the delay fits the data. H_1: The model for the delay does not fit the data.	B1 B1	Do not allow hypotheses of the form "Data fit model" o.e.

Obs'd frequency	160	40	13	9	3
Exp'd frequency	142.23	52.32	19.25	7.08	4.12

Merge last 2 cells: Obs 12 Exp 11.2	M1	
X^2 = 2.2202 + 2.9010 + 2.0292 + 0.0571	M1	Calculation of X^2.
= 7.207(5)	A1	cao. If not merged, X^2 = 7.975(5…)
Refer to χ^2_2.	M1	No ft if wrong. Allow correct dof (= cells – 2) from wrongly grouped table and ft. Allow any value from tables for c's dof.
Upper 2.5% point is 7.378.	A1	c.a.o. Upper 2.5% point for c's dof. No ft from here if wrong. P($X^2 > 7.2075$) = 0.0272.

Question			Answer	Marks	Guidance
			$7.207 < 7.378$ ∴ Not Significant. Suggests it is reasonable to suppose the model fits the data.	A1 A1 **[9]**	ft only c's test statistic. ft only c's test statistic. "Non-assertive" conclusion in words (+ context). Do not allow "Data fit model" o.e.
3	**(b)**	**(i)**	A paired test is used in this context in order to eliminate differences between health authorities.	E1 **[1]**	oe
3	**(b)**	**(ii)**			

Diff	11	26	–15	4	–9	–1	23	5
Rank	5	8	6	2	4	1	7	3

	M1	For differences. ZERO in this section if differences not used.
	M1	For ranks.
	A1	ft from here if ranks wrong.
$W_- = 1 + 4 + 6 = 11$	B1	(or $W_+ = 2 + 3 + 5 + 7 + 8 = 25$)
Refer to tables of Wilcoxon paired (/single sample) statistic for $n = 8$.	M1	No ft from here if wrong.
Lower 5% tail is 5 (or upper is 31 if 25 used).	A1	ie a 2-tail test. No ft from here if wrong.
$11 > 5$ ∴ Result is not significant.	A1	ft only c's test statistic.
No evidence to suggest a difference between the incidences of myocardial infarction in men and women on the whole.	A1 **[8]**	ft only c's test statistic. "Non-assertive" conclusion in context to include "on the whole" oe.
4	**(i)**		$\mathrm{P}(R < r) = k\pi r^2$ $\mathrm{P}(R < a) = 1 \quad \therefore k = \frac{1}{\pi a^2}$ $\therefore \mathrm{P}(R < r) = \frac{\pi r^2}{\pi a^2} = \frac{r^2}{a^2}$. Thus $\mathrm{F}(r) = \frac{r^2}{a^2}$ (for $0 \le r \le a$).	M1 M1 A1 **[3]**	Formulate probability proportional to area in terms of "k". Find k. IF M0M0, allow SC B1 for $\mathrm{P}(R < r) = \frac{\pi r^2}{\pi a^2} = \ldots$ www Convincingly shown; ANSWER GIVEN. Condone omission of $r < 0$ and/or $r > a$.
4	**(ii)**		For $0 \le r \le a$, $\mathrm{f}(r) = \frac{\mathrm{d}}{\mathrm{d}r}\mathrm{F}(r) = \frac{2r}{a^2}$.	M1 A1 **[2]**	 Condone omission of $r < 0$ and/or $r > a$.

Question			Answer	Marks	Guidance
4	(iii)		$\mathrm{E}(R) = \int_0^a r\frac{2r}{a^2}\,\mathrm{d}r$	M1	Correct integral with limits (which may be implied subsequently).
			$= \left[\frac{2r^3}{3a^2}\right]_0^a$	A1	Correctly integrated.
			$= \frac{2a}{3}$	A1	Limits used. Accept unsimplified form.
			$\mathrm{E}(R^2) = \int_0^a r^2\frac{2r}{a^2}\,\mathrm{d}r$	M1	Correct integral with limits (which may be implied subsequently).
			$= \left[\frac{2r^4}{4a^2}\right]_0^a = \frac{a^2}{2}$	A1	Correctly integrated and limits used. Accept unsimplified form.
			$\mathrm{Var}(R) = \frac{a^2}{2} - \left(\frac{2a}{3}\right)^2 = \frac{9a^2 - 8a^2}{18} = \frac{a^2}{18}$	M1 A1	Use of $\mathrm{Var}(R) = \mathrm{E}(R^2) - \mathrm{E}(R)^2$ Convincingly shown; ANSWER GIVEN. Require sight of both terms expressed with a common denominator.
				[7]	
4	(iv)		$\overline{R} \sim$ (approx) $\mathrm{N}\left(\frac{2}{3}\times 22.5 = 15,\ \frac{22.5^2}{18\times 100} = 0.28125\right)$	B1 B1 B1	Normal. Mean. ft c's E(R) (>0) with $a = 22.5$. Variance. cao ($= 0.5303(3)^2$) Accept unsimplified form.
				[3]	
4	(v)		**EITHER** can argue that 13.87 is more than 2 SD's from the Mean (15).	M1	Allow 1.96 SD's, but not 1.984.
			$15 - 2\sqrt{0.28125} = 13.93(9)$		1.96 gives 13.96(1). A 95% C.I. is (12.831, 14.909).
			must refer to $\mathrm{SD}(\overline{R})$, not $\mathrm{SD}(R)$	M1	Must see explicit evidence for this.
			i.e. outlier $\Rightarrow$ doubt.	A1	ft c's mean.
				[3]	
			OR more formally like a significance test: refer to N(0,1)	M1	
			$\frac{13.87 - 15}{\sqrt{0.28125}} = -2.131$, sig at (eg) 5%	M1	Could imply first M. $\mathrm{P}(\lvert Z\rvert > 2.131) = 0.0332$.
			$\Rightarrow$ doubt.	A1	ft c's mean.

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