

Tuesday 17 January 2012 – Morning

A2 GCE MATHEMATICS (MEI)

4754A Applications of Advanced Mathematics (C4) Paper A

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:

- Printed Answer Book 4754A
- MEI Examination Formulae and Tables (MF2)

Other materials required:

- Scientific or graphical calculator

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.
- This paper will be followed by **Paper B: Comprehension**.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

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Section A (36 marks)

1 Express $\frac{x+1}{x^2(2x-1)}$ in partial fractions. [5]

2 Solve, correct to 2 decimal places, the equation $\cot 2\theta = 3$ for $0^\circ \leq \theta \leq 180^\circ$. [4]

3 Express $3 \sin x + 2 \cos x$ in the form $R \sin(x + \alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Hence find, correct to 2 decimal places, the coordinates of the maximum point on the curve $y = f(x)$, where

$$f(x) = 3 \sin x + 2 \cos x, \quad 0 \leq x \leq \pi. \quad [7]$$

4 (i) Complete the table of values for the curve $y = \sqrt{\cos x}$.

x	0	$\frac{\pi}{8}$	$\frac{\pi}{4}$	$\frac{3\pi}{8}$	$\frac{\pi}{2}$
y		0.9612	0.8409		

Hence use the trapezium rule with strip width $h = \frac{\pi}{8}$ to estimate the value of the integral $\int_0^{\frac{\pi}{2}} \sqrt{\cos x} \, dx$, giving your answer to 3 decimal places. [3]

Fig. 4 shows the curve $y = \sqrt{\cos x}$ for $0 \leq x \leq \frac{\pi}{2}$.

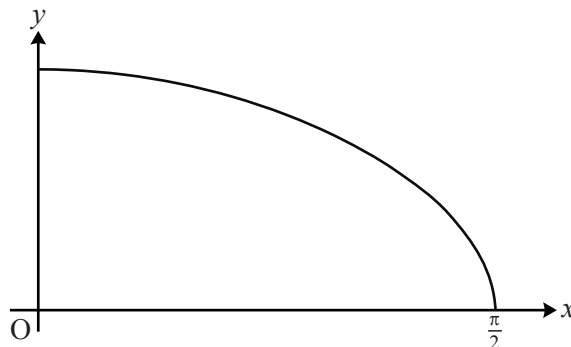


Fig. 4

(ii) State, with a reason, whether the trapezium rule with a strip width of $\frac{\pi}{16}$ would give a larger or smaller estimate of the integral. [1]

5 Verify that the vector $2\mathbf{i} - \mathbf{j} + 4\mathbf{k}$ is perpendicular to the plane through the points $A(2, 0, 1)$, $B(1, 2, 2)$ and $C(0, -4, 1)$. Hence find the cartesian equation of the plane. [5]

6 Given the binomial expansion $(1 + qx)^p = 1 - x + 2x^2 + \dots$, find the values of p and q . Hence state the set of values of x for which the expansion is valid. [6]

7 Show that the straight lines with equations $\mathbf{r} = \begin{pmatrix} 4 \\ 2 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} -1 \\ 4 \\ 9 \end{pmatrix} + \mu \begin{pmatrix} -1 \\ 1 \\ 3 \end{pmatrix}$ meet.

Find their point of intersection. [5]

Section B (36 marks)

- 8 Fig. 8 shows a cross-section of a car headlight whose inside reflective surface is modelled, in suitable units, by the curve

$$x = 2t^2, y = 4t, \quad -\sqrt{2} \leq t \leq \sqrt{2}.$$

$P(2t^2, 4t)$ is a point on the curve with parameter t . TS is the tangent to the curve at P , and PR is the line through P parallel to the x -axis. Q is the point $(2, 0)$. The angles that PS and QP make with the positive x -direction are θ and ϕ respectively.

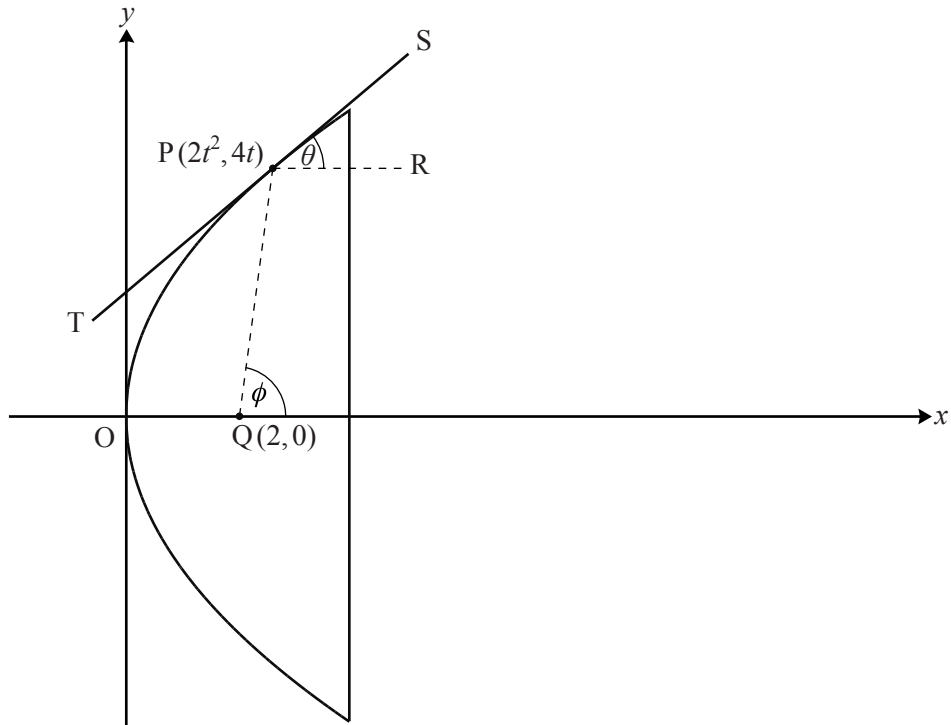


Fig. 8

- (i) By considering the gradient of the tangent TS , show that $\tan \theta = \frac{1}{t}$. [3]
- (ii) Find the gradient of the line QP in terms of t . Hence show that $\phi = 2\theta$, and that angle TPQ is equal to θ . [8]

[The above result shows that if a lamp bulb is placed at Q , then the light from the bulb is reflected to produce a parallel beam of light.]

The inside surface of the headlight has the shape produced by rotating the curve about the x -axis.

- (iii) Show that the curve has cartesian equation $y^2 = 8x$. Hence find the volume of revolution of the curve, giving your answer as a multiple of π . [7]

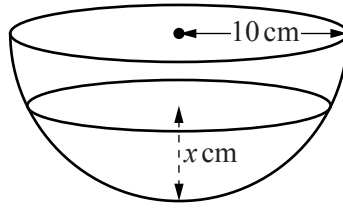


Fig. 9

Fig. 9 shows a hemispherical bowl, of radius 10 cm, filled with water to a depth of x cm. It can be shown that the volume of water, $V \text{ cm}^3$, is given by

$$V = \pi(10x^2 - \frac{1}{3}x^3).$$

Water is poured into a leaking hemispherical bowl of radius 10 cm. Initially, the bowl is empty. After t seconds, the volume of water is changing at a rate, in $\text{cm}^3 \text{ s}^{-1}$, given by the equation

$$\frac{dV}{dt} = k(20 - x),$$

where k is a constant.

(i) Find $\frac{dV}{dx}$, and hence show that $\pi x \frac{dx}{dt} = k$. [4]

(ii) Solve this differential equation, and hence show that the bowl fills completely after T seconds, where $T = \frac{50\pi}{k}$. [5]

Once the bowl is full, the supply of water to the bowl is switched off, and water then leaks out at a rate of $kx \text{ cm}^3 \text{ s}^{-1}$.

(iii) Show that, t seconds later, $\pi(20 - x) \frac{dx}{dt} = -k$. [3]

(iv) Solve this differential equation.

Hence show that the bowl empties in $3T$ seconds. [6]

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