RECOGNISING ACHIEVEMENT

GCE

Mathematics (MEI)

Advanced GCE

Unit **4758:** Differential Equations

Mark Scheme for June 2011

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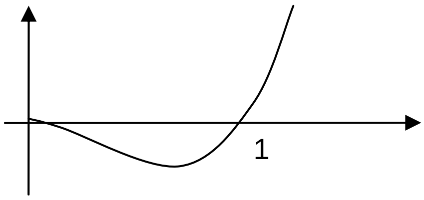
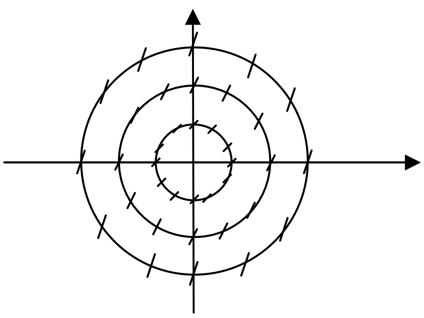
1(i)	$\lambda^2+4\lambda+3=0$	M1	Auxiliary equation	
	$\lambda=-1$ or -3	A1		
	CF $Ae^{-t}+Be^{-3t}$	F1	CF for their roots	
	PI $y=a\cos 2t+b\sin 2t$	B1		
	$\dot{y}=-2a\sin 2t+2b\cos 2t$	M1	Differentiate twice and substitute	
	$\ddot{y}=-4a\cos 2t-4b\sin 2t$			
	$-4a\cos 2t-4b\sin 2t-8a\sin 2t+8b\cos 2t+3a\cos 2t+3b\sin 3t=13\cos 2t$	M1	Compare coefficients	
	$8b-a=13$ $-b-8a=0$	A1		
	$a=-\frac{1}{5},\ b=\frac{8}{5}$	A1		
	GS $y=\frac{1}{5}(8\sin 2t-\cos 2t)+Ae^{-t}+Be^{-3t}$	F1	PI + CF with two arbitrary constants	
				9
(ii)	$t=0,\ y=0\Rightarrow 0=-\frac{1}{5}+A+B$	M1	Use condition	
	$\dot{y}=\frac{1}{5}(16\cos 2t+2\sin 2t)-Ae^{-t}-3Be^{-3t}$	M1 F1	Differentiate	
	$t=0,\ \dot{y}=0\Rightarrow 0=\frac{16}{5}-A-3B$	M1	Use condition	
	$\Rightarrow A=-\frac{13}{10},\ B=\frac{3}{2}$	A1		
	$y=\frac{1}{5}(8\sin 2t-\cos 2t)-\frac{13}{10}e^{-t}+\frac{3}{2}e^{-3t}$	A1	Cao	
				6
(iii)	If $z=y+c$, differentiating (*) gives new DE	M1	Recognise derivative	
	and has 3 arbitrary constants so must be GS	A1		
	or			
	Integrating gives (*) with $+k$ on RHS	*M1*		
	PI will be previous PI $+\frac{1}{3}k$, CF as before, so GS $y+c$	*A1*		
	SC1 for showing that correct *y* from (i) + *c* satisfies new DE			2
(iv)	$z=\frac{1}{5}(8\sin 2t-\cos 2t)+De^{-t}+Ee^{-3t}+c$			
	$t=0,\ z=2\Rightarrow 2=-\frac{1}{5}+D+E+c$	M1	Use condition	
	$\dot{z}=\frac{1}{5}(16\cos 2t+2\sin 2t)-De^{-t}-3Ee^{-3t}$	F1	Derivative	
	$t=0,\ \dot{z}=0\Rightarrow 0=\frac{16}{5}-D-3E$	M1	Use condition	
	$\ddot{z}=\frac{1}{5}(-32\sin 2t+4\cos 2t)+De^{-t}+9Ee^{-3t}$	F1	Second derivative: condone, for this mark only, +*c* appearing	
	$t=0,\ \ddot{z}=13\Rightarrow 13=\frac{4}{5}+D+9E$	M1	Use condition	
	$D=-\frac{13}{10},\ E=\frac{3}{2},\ c=2$	B1		
	$z=\frac{1}{5}(8\sin 2t-\cos 2t)-\frac{13}{10}e^{-t}+\frac{3}{2}e^{-3t}+2$	A1	Cao	
				7

2(a)(i)	$I = \exp(\int -\frac{2}{x}\,\mathrm{d}x)$	M1	Attempt integrating factor	
	$= \exp(-2\ln x)$	A1		
	$= x^{-2}$	A1		
	$x^{-2}\frac{\mathrm{d}y}{\mathrm{d}x} - 2x^{-3}y = x^{-3/2}$	M1	Multiply both sides by IF	
	$\frac{\mathrm{d}}{\mathrm{d}x}(x^{-2}y) = x^{-3/2}$	M1		
	$x^{-2}y = -2x^{-1/2} + A$	M1	Integrate both sides	
		A1		
	$y = -2x^{3/2} + Ax^2$	F1	Must divide constant	8
(ii)	$0 = -2 + A$	M1		
	$y = 2x^2 - 2x^{3/2}$	A1		2
(iii)	$x \to 0,\ y \to 0$	F1		
	$\frac{\mathrm{d}y}{\mathrm{d}x} = 4x - 3x^{1/2} = 0 \Leftrightarrow x = \frac{9}{16}$ (as $x > 0$)	M1		
	$x \to 0,\ \frac{\mathrm{d}y}{\mathrm{d}x} \to 0$	F1		
		B1	Behaviour at origin	
		B1	Through (1,0) and shape for $x > 1$	
		B1	Stationary point at $\left(\frac{9}{16}, -\frac{27}{128}\right)$	6
(b)(i)	Circle centre origin	B1		
	Radius 1	B1		2
(ii)		B1	One isocline correct	
		B1	All three isoclines correct	
		B1	Reasonably complete and accurate direction indicators	3
(iii)		B1	Solution curve	1
(iv)		B1	Solution curve	
		B1	Zero gradient at origin	2

3(a)(i)	N2L: $ma = -2k^2 x$	M1		
	$2v\dfrac{dv}{dx} = -2k^2 x$	M1	Acceleration $= v\dfrac{dv}{dx}$	
	$v\dfrac{dv}{dx} = -k^2 x$	E1		
				3
(ii)	$\int v dv = \int -k^2 x dx$	M1	Separate and integrate	
	$\frac{1}{2}v^2 = -\frac{1}{2}k^2 x^2 + A$	A1	LHS	
		A1	RHS	
	$x = a,\ v = 0 \Rightarrow A = \frac{1}{2}k^2 a^2$	M1	Use condition	
	$v^2 = k^2(a^2 - x^2)$	A1		
	So for $v < 0$, $\dfrac{dx}{dt} = -k\sqrt{a^2 - x^2}$	E1		
				6
(iii)	$\int \dfrac{1}{\sqrt{a^2 - x^2}} dx = \int -k dt$	M1	Separate and integrate	
	$\arcsin \frac{x}{a} + B = -kt$	A1	LHS	
		A1	RHS	
	$x = a,\ t = 0 \Rightarrow B = -\frac{1}{2}\pi$	M1	Use condition	
	$x = a\sin(\frac{1}{2}\pi - kt) = a\cos kt$	A1	Either form	
				5
(b)(i)	$\int \omega d\omega = \int -9\sin\theta d\theta$	M1	Separate and integrate	
	$\frac{1}{2}\omega^2 = 9\cos\theta + C$	A1	LHS	
		A1	RHS	
	$\theta = \frac{1}{3}\pi,\ \omega = 0 \Rightarrow C = -\frac{9}{2}$	M1	Use condition	
	So $\omega^2 = 9(2\cos\theta - 1)$	A1		
	$\dfrac{d\theta}{dt} = -3\sqrt{2\cos\theta - 1}$ (decreasing)	E1		
				6
(ii)	$\theta = \frac{1}{3}\pi \Rightarrow \dot{\theta} = 0$	M1		
	So estimate $= \frac{1}{3}\pi + 0 = \frac{1}{3}\pi$	A1		
	The algorithm will keep giving $\theta = \frac{1}{3}\pi$	B1		
	but θ is not constant so not useful	B1		
				4

4(i)	$y=-\frac{1}{2}\dot{x}-\frac{3}{2}x+\frac{3}{2}t$	M1		
	$\dot{y}=-\frac{1}{2}\ddot{x}-\frac{3}{2}\dot{x}+\frac{3}{2}$	M1		
	$-\frac{1}{2}\ddot{x}-\frac{3}{2}\dot{x}+\frac{3}{2}=2x+(-\frac{1}{2}\dot{x}-\frac{3}{2}x+\frac{3}{2}t)+t+2$	M1	Eliminate y	
		M1	Eliminate $\dot{y}$	
	$\ddot{x}+2\dot{x}+x=-5t-1$	E1		5
(ii)	$\lambda^2+2\lambda+1=0$	M1	Auxiliary equation	
	$\lambda=-1$ (repeated)	A1	Root	
	CF: $(A+Bt)\mathrm{e}^{-t}$	F1	CF for their root(s) (with two constants)	
	PI: $x=at+b$	B1		
	$\dot{x}=a,\ \ddot{x}=0$			
	In DE: $0+2a+at+b=-5t-1$	M1	Differentiate and substitute	
	$a=-5$			
	$2a+b=-1$	M1	Compare and solve	
	$a=-5,\ b=9$	A1		
	GS: $x=9-5t+(A+Bt)\mathrm{e}^{-t}$	F1	GS = PI + CF with two arbitrary constants	8
(iii)	$y=-\frac{1}{2}\dot{x}-\frac{3}{2}x+\frac{3}{2}t$	M1		
	$=-\frac{1}{2}[-5+B\mathrm{e}^{-t}-(A+Bt)\mathrm{e}^{-t}]$	M1	Differentiate (product rule)	
	$-\frac{3}{2}[9-5t+(A+Bt)\mathrm{e}^{-t}]+\frac{3}{2}t$	M1	Substitute	
	$=9t-11-(A+\frac{1}{2}B+Bt)\mathrm{e}^{-t}$	A1		4
(iv)	$t=0,\ x=9\Rightarrow A=0$	M1	Use condition	
	$t=0,\ y=0\Rightarrow 0=-11-\frac{1}{2}B\Rightarrow B=-22$	M1	Use condition	
	$x=9-5t-22t\mathrm{e}^{-t}$	A1		
	$y=9t-11+(11+22t)\mathrm{e}^{-t}$	A1		4
(v)	$\mathrm{e}^{-t}\to 0$	M1		
	$x\approx 9-5t$	F1		
	$y\approx 9t-11$	F1		3

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