OCR RECOGNISING ACHIEVEMENT

GCE

Mathematics (MEI)

Advanced Subsidiary GCE

Unit **4755:** Further Concepts for Advanced Mathematics

Mark Scheme for June 2011

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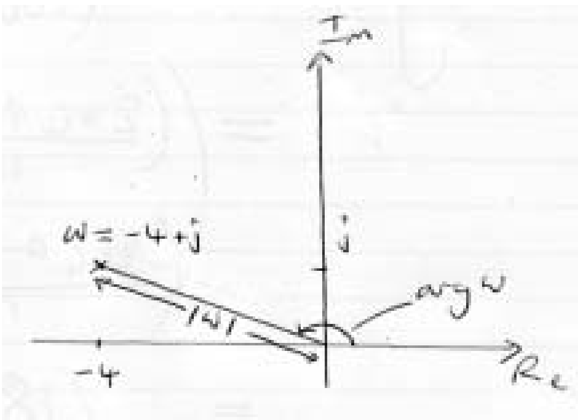
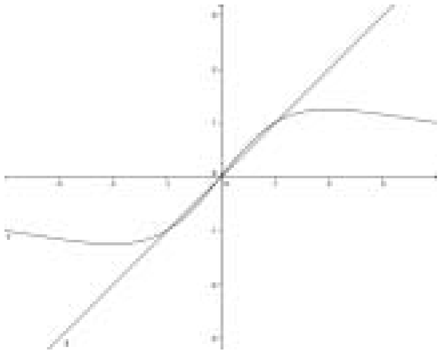
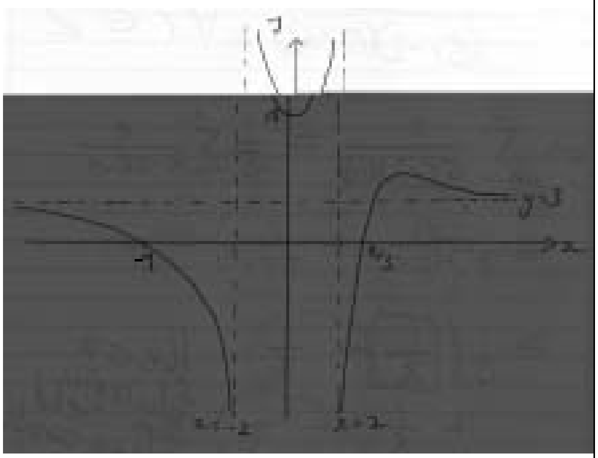
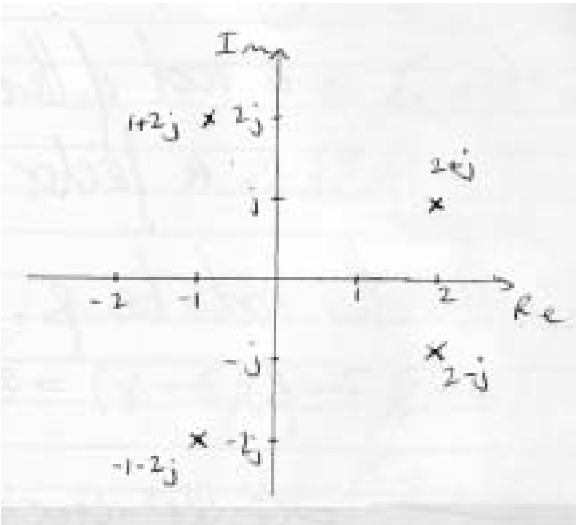
Qu	Answer	Mark	Comment
Section A			
1(i)	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	B1	Accept expressions in sin and cos
1(ii)	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	B1	
1(iii)	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	M1 A1ft	Ans (ii) x Ans (i) attempt evaluation
1(iv)	Reflection in the *x* axis	B1 **[5]**	
2(i)	$\frac{z+w}{w} = \frac{-1-\text{j}}{-4+\text{j}} \times \frac{-4-\text{j}}{-4-\text{j}}$ $= \frac{3+5\text{j}}{17} = \frac{3}{17} + \frac{5}{17}\text{j}$	M1 A1 A1 **[3]**	Multiply top and bottom by -4 - j Denominator = 17 Correct numerators
2(ii)	$\lvert w \rvert = \sqrt{17}$ $\arg w = \pi - \arctan\frac{1}{4} = 2.90$ $w = \sqrt{17}\left(\cos 2.90 + \text{j}\sin 2.90\right)$	B1 B1 B1 **[3]**	 Not degrees c.a.o. Accept $\left(\sqrt{17},\ 2.90\right)$ Accept 166 degrees
2(iii)		B1 B1 **[2]**	Correct position Mod w and Arg w correctly shown
3	$\alpha + \beta + \gamma = 4 = -p$ $p = -4$ $\left(\alpha+\beta+\gamma\right)^2 = \alpha^2 + \beta^2 + \gamma^2 + 2\left(\alpha\beta + \alpha\gamma + \beta\gamma\right)$ $\Rightarrow 16 = 6 + 2q$ $\Rightarrow q = 5$	M1 A1 M1 A1 A1 **[5]**	May be implied Attempt to use $\left(\alpha+\beta+\gamma\right)^2$ o.e. Correct c.a.o.

4	$\frac{5x}{x^2+4}<x$ $\Rightarrow 5x<x^3+4x$ $\Rightarrow 0<x^3-x$ $\Rightarrow 0<x(x+1)(x-1)$ $\Rightarrow x>1,\ \ -1<x<0$ 	M1* A1 A1 M1dep* A1 A1 **[6]**	Method attempted towards factorisation to find critical values $x = 0$ $x = 1$, $x = -1$ Valid method leading to required intervals, graphical or algebraic $x>1$ $-1<x<0$ SC B2 No valid working seen $x>1$ $-1<x<0$
5	$\sum_{r=1}^{20}\frac{1}{(3r-1)(3r+2)}\equiv\frac{1}{3}\sum_{r=1}^{20}\left[\frac{1}{3r-1}-\frac{1}{3r+2}\right]$ $=\frac{1}{3}\left[\left(\frac{1}{2}-\frac{1}{5}\right)+\left(\frac{1}{5}-\frac{1}{8}\right)+....+\left(\frac{1}{59}-\frac{1}{62}\right)\right]$ $=\frac{1}{3}\left(\frac{1}{2}-\frac{1}{62}\right)=\frac{5}{31}$	M1 A1 A1 M1 A1 **[5]**	Attempt to use identity – may be implied Correct use of 1/3 seen Terms in full (at least first and last) Attempt at cancelling c.a.o.

6	When $n = 1$, $\frac{1}{4}n^2(n+1)^2 = 1$, so true for $n = 1$	B1	
	Assume true for $n = k$ $\sum_{r=1}^{k} r^3 = \frac{1}{4}k^2(k+1)^2$	E1	Assume true for k
	$\Rightarrow \sum_{r=1}^{k+1} r^3 = \frac{1}{4}k^2(k+1)^2 + (k+1)^3$	M1	Add $(k+1)$th term to both sides
	$= \frac{1}{4}(k+1)^2[k^2 + 4(k+1)]$ $= \frac{1}{4}(k+1)^2[k^2 + 4k + 4]$	M1	Factor of $\frac{1}{4}(k+1)^2$
	$= \frac{1}{4}(k+1)^2(k+2)^2$ $= \frac{1}{4}(k+1)^2((k+1)+1)^2$	A1	c.a.o. with correct simplification
	But this is the given result with $k + 1$ replacing k. Therefore if it is true for k it is true for $k + 1$.	E1	Dependent on A1 and previous E1
	Since it is true for $n = 1$, it is true for $n = 1, 2, 3$ and so true for all positive integers.	E1 **[7]**	Dependent on B1 and previous E1 and correct presentation
			Section A Total: 36

Section B			
7(i)	$(0, 18)$	B1	
	$(-9, 0)$, $\left(\frac{8}{3}, 0\right)$	B1 B1 **[3]**	
7(ii)	$x = 2$, $x = -2$ and $y = 3$	B1 B1 B1 **[3]**	
7(iii)	Large positive x, $y \to 3^{+}$ from above Large negative x, $y \to 3^{-}$ from below	B1 B1	
	(e.g. consider $x = 100$, or convincing algebraic argument)	M1 **[3]**	Must show evidence of working
7(iv)		B1 B1 B1 **[3]**	3 branches correct Asymptotes correct and labelled Intercepts correct and labelled

8(i)	Because a cubic can only have a maximum of two complex roots, which must form a conjugate pair.	E1 **[1]**	.
8(ii)	$2+\text{j},\ -1-2\text{j}$	B1 B1	
	$\text{P}(z)=(z-(2-\text{j}))(z-(2+\text{j}))(z-(-1+2\text{j}))(z-(-1-2\text{j}))$	M1	Use of factor theorem
	$=((z-2)^2+1)((z+1)^2+4)$	M1	Attempt to multiply out factors
	$=(z^2-4z+5)(z^2+2z+5)$ $=z^4-2z^3+2z^2-10z+25$	A4	-1 for each incorrect coefficient
	OR		
	$\alpha+\beta+\gamma+\delta=2\Rightarrow a=-2$ $\alpha\beta+\alpha\gamma+\alpha\delta+\beta\gamma+\beta\delta+\gamma\delta=2\Rightarrow b=2$ $\alpha\beta\gamma+\alpha\beta\delta+\alpha\gamma\delta+\beta\gamma\delta=10\Rightarrow c=-10$ $\alpha\beta\gamma\delta=25\Rightarrow d=25$ $\Rightarrow \text{P}(z)=z^4-2z^3+2z^2-10z+25$	M2 B1 A3	M1 for attempt to use all 4 root relationships. M2 for all correct a = -2 b, c, d correct -1 for each incorrect -1 for P(z) not explicit, following A4 or B1A3
		[8]	
8(iii)		B1	All correct with annotation on axes or labels
	$\|z\|=\sqrt{5}$	B1 **[2]**	

Qu	Answer	Mark	Comment
Section B (continued)			
9(i)	$\mathbf{M}=\begin{pmatrix}2 & -1\\3 & k\end{pmatrix}$	B2 **[2]**	- 1 each error
9(ii)	$\mathbf{M}^{-1}$ does not exist for $2k+3=0$	M1	May be implied
	$k=\frac{-3}{2}$	A1	
	$\mathbf{M}^{-1}=\frac{1}{2k+3}\begin{pmatrix}k & 1\\-3 & 2\end{pmatrix}$	B1	Correct inverse
	$\frac{1}{13}\begin{pmatrix}5 & 1\\-3 & 2\end{pmatrix}\begin{pmatrix}1\\21\end{pmatrix}$	M1	Attempt to pre-multiply by their inverse
	$=\begin{pmatrix}2\\3\end{pmatrix}$	A1ft A1	Correct matrix multiplication c.a.o.
	$\Rightarrow x=2,\ y=3$	A1ft	At least one correct
		[7]	
9(iii)	There are no unique solutions	B1 **[1]**	
9(iv)	(A) Lines intersect (B) Lines parallel (C) Lines coincident	B1 B1 B1 **[3]**	
			Section B Total: 36
			Total: 72

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