

**ADVANCED GCE
MATHEMATICS (MEI)**

Further Methods for Advanced Mathematics (FP2)

4756

Candidates answer on the answer booklet.

OCR supplied materials:

- 8 page answer booklet (sent with general stationery)
- MEI Examination Formulae and Tables (MF2)

Other materials required:

- Scientific or graphical calculator

**Monday 20 June 2011
Morning**

Duration: 1 hour 30 minutes



INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet. Please write clearly and in capital letters.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions in Section A and **one** question from Section B.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

Section A (54 marks)

Answer all the questions

- 1 (a) A curve has polar equation $r = a(1 - \sin \theta)$, where $a > 0$ and $0 \leq \theta < 2\pi$.
- (i) Sketch the curve. [2]
- (ii) Find, in an exact form, the area of the region enclosed by the curve. [7]

(b) (i) Find, in an exact form, the value of the integral $\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{1}{1+4x^2} dx$. [3]

(ii) Find, in an exact form, the value of the integral $\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{1}{(1+4x^2)^{\frac{3}{2}}} dx$. [6]

- 2 (a) Use de Moivre's theorem to find expressions for $\sin 5\theta$ and $\cos 5\theta$ in terms of $\sin \theta$ and $\cos \theta$.

Hence show that, if $t = \tan \theta$, then

$$\tan 5\theta = \frac{t(t^4 - 10t^2 + 5)}{5t^4 - 10t^2 + 1}. \quad [6]$$

- (b) (i) Find the 5th roots of $-4\sqrt{2}$ in the form $re^{j\theta}$, where $r > 0$ and $0 \leq \theta < 2\pi$. [4]

These 5th roots are represented in the Argand diagram, in order of increasing θ , by the points A, B, C, D, E.

- (ii) Draw the Argand diagram, making clear which point is which. [2]

The mid-point of AB is the point P which represents the complex number w .

- (iii) Find, in exact form, the modulus and argument of w . [3]

- (iv) w is an n th root of a real number a , where n is a positive integer. State the least possible value of n and find the corresponding value of a . [3]

- 3 (i) Find the value of k for which the matrix

$$\mathbf{M} = \begin{pmatrix} 1 & -1 & k \\ 5 & 4 & 6 \\ 3 & 2 & 4 \end{pmatrix}$$

does not have an inverse.

Assuming that k does not take this value, find the inverse of \mathbf{M} in terms of k . [7]

- (ii) In the case $k = 3$, evaluate

$$\mathbf{M} \begin{pmatrix} -3 \\ 3 \\ 1 \end{pmatrix}. \quad [2]$$

- (iii) State the significance of what you have found in part (ii). [2]

- (iv) Find the value of t for which the system of equations

$$\begin{aligned} x - y + 3z &= t \\ 5x + 4y + 6z &= 1 \\ 3x + 2y + 4z &= 0 \end{aligned}$$

has solutions. Find the general solution in this case and describe the solution geometrically. [7]

Section B (18 marks)

Answer one question

Option 1: Hyperbolic functions

- 4 (i) Given that $\cosh y = x$, show that $y = \pm \ln(x + \sqrt{x^2 - 1})$ and that $\operatorname{arcosh} x = \ln(x + \sqrt{x^2 - 1})$. [7]

- (ii) Find $\int_{\frac{4}{5}}^1 \frac{1}{\sqrt{25x^2 - 16}} dx$, expressing your answer in an exact logarithmic form. [5]

- (iii) Solve the equation

$$5 \cosh x - \cosh 2x = 3,$$

giving your answers in an exact logarithmic form. [6]

Option 2: Investigation of curves

This question requires the use of a graphical calculator.

5 In this question, you are required to investigate the curve with equation

$$y = x^m(1 - x)^n, \quad 0 \leq x \leq 1,$$

for various positive values of m and n .

(i) On separate diagrams, sketch the curve in each of the following cases.

(A) $m = 1, n = 1,$

(B) $m = 2, n = 2,$

(C) $m = 2, n = 4,$

(D) $m = 4, n = 2.$

[4]

(ii) What feature does the curve have when $m = n$?

What is the effect on the curve of interchanging m and n when $m \neq n$?

[2]

(iii) Describe how the x -coordinate of the maximum on the curve varies as m and n vary. Use calculus to determine the x -coordinate of the maximum. [6]

(iv) Find the condition on m for the gradient to be zero when $x = 0$. State a corresponding result for the gradient to be zero when $x = 1$. [2]

(v) Use your calculator to investigate the shape of the curve for large values of m and n . Hence conjecture what happens to the value of the integral $\int_0^1 x^m(1 - x)^n dx$ as m and n tend to infinity. [2]

(vi) Use your calculator to investigate the shape of the curve for small values of m and n . Hence conjecture what happens to the shape of the curve as m and n tend to zero. [2]

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