OCR RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI) 4756

Further Methods for Advanced Mathematics (FP2)

Candidates answer on the answer booklet.

OCR supplied materials:
- 8 page answer booklet (sent with general stationery)
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Monday 20 June 2011
Morning

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet. Please write clearly and in capital letters.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions in Section A and **one** question from Section B.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

Section A (54 marks)

Answer all the questions

1 **(a)** A curve has polar equation $r = a(1 - \sin\theta)$, where $a > 0$ and $0 \leqslant \theta < 2\pi$.

(i) Sketch the curve. **[2]**

(ii) Find, in an exact form, the area of the region enclosed by the curve. **[7]**

(b) **(i)** Find, in an exact form, the value of the integral $\displaystyle\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{1}{1+4x^2}\,\mathrm{d}x$. **[3]**

(ii) Find, in an exact form, the value of the integral $\displaystyle\int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{1}{\left(1+4x^2\right)^{\frac{3}{2}}}\,\mathrm{d}x$. **[6]**

2 **(a)** Use de Moivre's theorem to find expressions for $\sin 5\theta$ and $\cos 5\theta$ in terms of $\sin\theta$ and $\cos\theta$.

Hence show that, if $t = \tan\theta$, then

$$\tan 5\theta = \frac{t(t^4 - 10t^2 + 5)}{5t^4 - 10t^2 + 1}.$$ **[6]**

(b) **(i)** Find the 5th roots of $-4\sqrt{2}$ in the form $r\mathrm{e}^{\mathrm{j}\theta}$, where $r > 0$ and $0 \leqslant \theta < 2\pi$. **[4]**

These 5th roots are represented in the Argand diagram, in order of increasing θ, by the points A, B, C, D, E.

(ii) Draw the Argand diagram, making clear which point is which. **[2]**

The mid-point of AB is the point P which represents the complex number w.

(iii) Find, in exact form, the modulus and argument of w. **[3]**

(iv) w is an nth root of a real number a, where n is a positive integer. State the least possible value of n and find the corresponding value of a. **[3]**

3 **(i)** Find the value of k for which the matrix

$$\mathbf{M} = \begin{pmatrix} 1 & -1 & k \\ 5 & 4 & 6 \\ 3 & 2 & 4 \end{pmatrix}$$

does not have an inverse.

Assuming that k does not take this value, find the inverse of $\mathbf{M}$ in terms of k. **[7]**

(ii) In the case $k = 3$, evaluate

$$\mathbf{M}\begin{pmatrix} -3 \\ 3 \\ 1 \end{pmatrix}.$$ **[2]**

(iii) State the significance of what you have found in part **(ii)**. **[2]**

(iv) Find the value of t for which the system of equations

$$\begin{aligned} x - y + 3z &= t \\ 5x + 4y + 6z &= 1 \\ 3x + 2y + 4z &= 0 \end{aligned}$$

has solutions. Find the general solution in this case and describe the solution geometrically. **[7]**

Section B (18 marks)

Answer one question

Option 1: Hyperbolic functions

4 **(i)** Given that $\cosh y = x$, show that $y = \pm \ln\left(x + \sqrt{x^2 - 1}\right)$ and that $\operatorname{arcosh} x = \ln\left(x + \sqrt{x^2 - 1}\right)$. **[7]**

(ii) Find $\displaystyle\int_{\frac{4}{5}}^{1} \frac{1}{\sqrt{25x^2 - 16}}\, dx$, expressing your answer in an exact logarithmic form. **[5]**

(iii) Solve the equation

$$5\cosh x - \cosh 2x = 3,$$

giving your answers in an exact logarithmic form. **[6]**

Option 2: Investigation of curves

This question requires the use of a graphical calculator.

5 In this question, you are required to investigate the curve with equation

$$y = x^m(1-x)^n, \qquad 0 \leqslant x \leqslant 1,$$

for various positive values of m and n.

(i) On separate diagrams, sketch the curve in each of the following cases.

(A) $m = 1, n = 1,$

(B) $m = 2, n = 2,$

(C) $m = 2, n = 4,$

(D) $m = 4, n = 2.$ **[4]**

(ii) What feature does the curve have when $m = n$?

What is the effect on the curve of interchanging m and n when $m \neq n$? **[2]**

(iii) Describe how the x-coordinate of the maximum on the curve varies as m and n vary. Use calculus to determine the x-coordinate of the maximum. **[6]**

(iv) Find the condition on m for the gradient to be zero when $x = 0$. State a corresponding result for the gradient to be zero when $x = 1$. **[2]**

(v) Use your calculator to investigate the shape of the curve for large values of m and n. Hence conjecture what happens to the value of the integral $\int_0^1 x^m(1-x)^n \, dx$ as m and n tend to infinity. **[2]**

(vi) Use your calculator to investigate the shape of the curve for small values of m and n. Hence conjecture what happens to the shape of the curve as m and n tend to zero. **[2]**