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RECOGNISING ACHIEVEMENT

GCE

Mathematics (MEI)

Advanced GCE

Unit **4777:** Numerical Computation

Mark Scheme for June 2011

Oxford Cambridge and RSA Examinations

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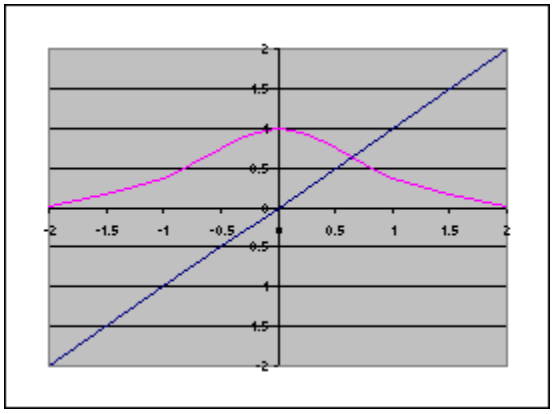
1(i)



r	x_r	
0	0.7	**[G2]**
1	0.612626	
2	0.687075	
3	0.623708	
4	0.677726	
5	0.631718	
6	0.670946	**[M1A1]**
7	0.637521	
8	0.666022	

- slow convergence

Derivative of $\exp(-x^2)$ is $-2x \exp(-x^2)$. Value at 0.6 (0.7) about -0.83 (-0.86). **[M1A1A1]**
Less than 1 in magnitude so converges, but not close to zero so slow. **[E1E1]**
[subtotal 9]

(ii) Multiply both sides by λ, then add (1 - λ) x to both sides. **[M1A1]**

λ	x_0	x_1	x_2	x_3	x_4	x_5	x_6	
0.4	0.7	0.6651	0.6561	0.6537	0.6531	0.6530	0.6529	
0.5	0.7	0.6563	0.6532	0.6529	0.6529	0.6529	0.6529	
0.6	0.7	0.6476	0.6535	0.6529	0.6529	0.6529	0.6529	**[M1A1A1]**

λ = 0.5 (to 1 dp) seems fastest **[A1]**
[subtotal 6]

(iii) Differentiate RHS, set to zero at x = α and solve for λ **[M1A1]**

Best λ evaluates to about 0.53978 **[B1]**
[subtotal 3]

(iv)

r	0	1	2	3	4	
x_r	0.7	0.652966	0.652919	0.652919	0.652919	
λ_r	0.538307	0.539778	0.53978	0.53978	0.53978	**[M1A1A1]**
Δx_r		-0.04703	-4.8E-05	-1.2E-10	0	
$\Delta x_{r+1}/\Delta x_r$			0.001011	2.47E-06	0	**[M1A1]**

Ratio of differences tending (rapidly) to zero so (much) faster than first order **[E1]**
[subtotal 6]
[TOTAL 24]

2(i) set up RHS as $a(f(-\alpha) + f(\alpha))$ (by symmetry) *(award same marks* **[M1A1]**

$f(x) = 1$: $2h = 2a$ hence $a = h$ *for solution without* **[M1A1]**

$f(x) = x$: $0 = 0$ *symmetry assumed)* **[A1]**

$f(x) = x^2$ $2h^3/3 = 2a\alpha^2$ hence $\alpha = h/\text{sqrt}(3)$ **[A1]**

$f(x) = x^3$ $0 = 0$ so no error **[A1]**

$f(x) = x^4$ $2h^5/5 = 2a\alpha^4 = 2h^5/9$ so local error of order h^5, global error h^4 **[A1E1E1]**

error is $8h^5/45$ ***[subtotal 10]***

(ii)

h	m	m-h/sqrt3	m+h/sqrt3	f(m-h/sqrt3)	f(m+h/sqrt3)	integral	diffs	
0.5	0.5	0.211325	0.788675	1.486525	1.646032	1.566278	*ratios*	**[M1A1]**
0.25	0.25	0.105662	0.394338	1.451022	1.544084			
0.25	0.75	0.605662	0.894338	1.602906	1.667272	1.566321	4.29E-05	**[M1A1]**
0.125	0.125	0.052831	0.197169	1.432762	1.481855			
0.125	0.375	0.302831	0.447169	1.515989	1.55962			
0.125	0.625	0.552831	0.697169	1.589056	1.625438			
0.125	0.875	0.802831	0.947169	1.649038	1.676832	1.566324	2.73E-06	**[A1]**
0.0625	0.0625	0.026416	0.098584	1.423521	1.448594		*0.06371*	
0.0625	0.1875	0.151416	0.223584	1.466573	1.490546			
0.0625	0.3125	0.276416	0.348584	1.507617	1.530218			
0.0625	0.4375	0.401416	0.473584	1.546196	1.567188			
0.0625	0.5625	0.526416	0.598584	1.581909	1.601085			
0.0625	0.6875	0.651416	0.723584	1.614408	1.631587			
0.0625	0.8125	0.776416	0.848584	1.643389	1.658417			
0.0625	0.9375	0.901416	0.973584	1.668594	1.681341	1.566324	1.71E-07	**[A1]**
							0.06275	

Ratio of differences very close to the theoretical 0.0625 for fourth order **[M1A1E1]**

[subtotal 9]

(iii) e.g.:

k	h	m	m-h/sqrt3	m+h/sqrt3	f(m-h/sqrt3)	f(m+h/sqrt3)	integral	
1.22	0.5	0.5	0.211325	0.788675	2.630873	3.373721	3.002297	
	0.25	0.25	0.105662	0.394338	2.480189	2.886405		
	0.25	0.75	0.605662	0.894338	3.162099	3.480932	3.002406	
	0.125	0.125	0.052831	0.197169	2.404721	2.610753		
	0.125	0.375	0.302831	0.447169	2.759934	2.95778		
	0.125	0.625	0.552831	0.697169	3.095848	3.271659		
	0.125	0.875	0.802831	0.947169	3.388775	3.529836	3.002413	
	0.0625	0.0625	0.026416	0.098584	2.367053	2.470074		
	0.0625	0.1875	0.151416	0.223584	2.545548	2.648271		
	0.0625	0.3125	0.276416	0.348584	2.72289	2.823568		
	0.0625	0.4375	0.401416	0.473584	2.896046	2.992924		
	0.0625	0.5625	0.526416	0.598584	3.061986	3.153343		
	0.0625	0.6875	0.651416	0.723584	3.21775	3.301937		
	0.0625	0.8125	0.776416	0.848584	3.360519	3.435994		
	0.0625	0.9375	0.901416	0.973584	3.487672	3.553039	3.002413	**[M3A2]**

k	1.2	1.3	1.21	1.23	1.22	
I	2.948	3.229	2.975	3.030	3.002	***[subtotal 5]***

[TOTAL 24]

3(i) Method A

h	x	y	k1	k2	new y
0.2	0	0	0.2	0.219089	0.219089
	0.2	0.219089	0.238251	0.255986	0.475075
	0.4	0.475075	0.273867	0.290655	0.76573
	0.6	0.76573	0.307619	0.3237	1.089429
	0.8	1.089429	0.339966	0.355495	1.444924
	1	1.444924	0.37121	0.386292	1.831216
	1.2	1.831216	0.401558	0.416269	2.247484
	1.4	2.247484	0.431161	0.445559	2.693043
	1.6	2.693043	0.460132	0.474262	3.167305
	1.8	3.167305	0.488561	0.502457	3.669763
	2	3.669763			

setup: **[M2]**

first run: **[A2]**

h	y(2)	diffs	ratio of diffs	
0.2	3.669763			
0.1	3.671640	0.001877		
0.05	3.672112	0.000473	0.251926	
0.025	3.672231	0.000119	0.250918	≈ 0.25 so 2nd order

further runs: **[A1A1A1]**

diffs + ratios: **[M1A1]**

[subtotal 9]

Method B

h	x	y	k1	k2	new y
0.2	0	0	0.2	0.236643	0.218322
	0.2	0.218322	0.238187	0.272507	0.473669
	0.4	0.473669	0.273764	0.306427	0.763764
	0.6	0.763764	0.307491	0.338896	1.086957
	0.8	1.086957	0.339821	0.370231	1.441983
	1	1.441983	0.371052	0.400651	1.827835
	1.2	1.827835	0.401389	0.430313	2.243686
	1.4	2.243686	0.430984	0.459333	2.688844
	1.6	2.688844	0.45995	0.487803	3.162721
	1.8	3.162721	0.488374	0.515794	3.664805
	2	3.664805			

setup: **[M2]**

first run: **[A2]**

h	y(2)	diffs	ratio of diffs	
0.2	3.664805			
0.1	3.670336	0.005531		
0.05	3.671778	0.001442	0.260722	
0.025	3.672146	0.000368	0.255408	≈ 0.25 so 2nd order

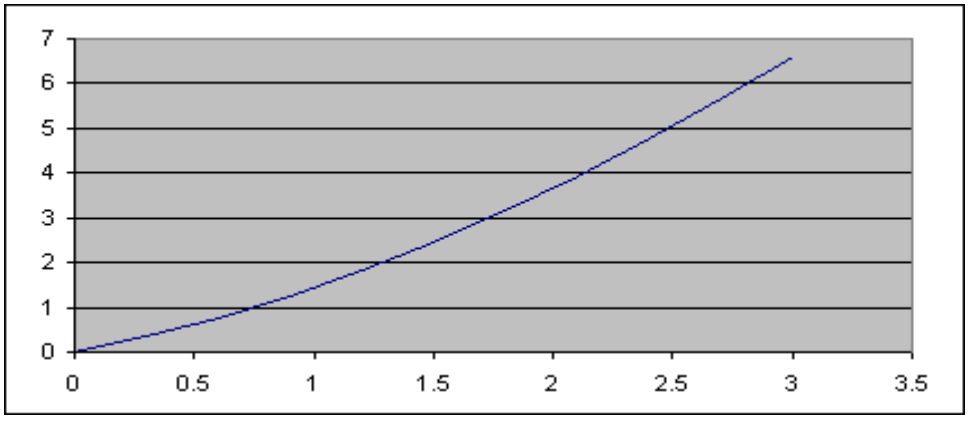
further runs: **[A1A1A1]**

diffs + ratios: **[M1A1]**

Differences (and hence errors) in Method B about 3 times those in method A **[M1E1]**

[subtotal 11]

(ii)



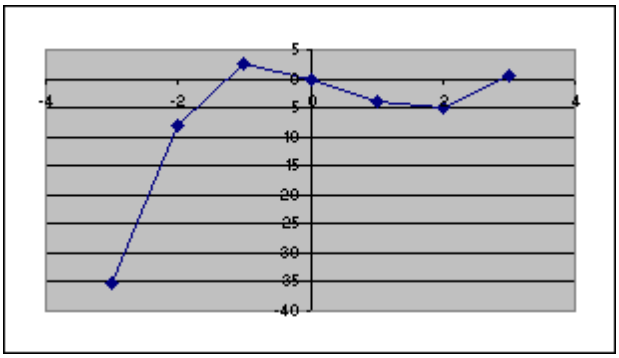
[G2]

Trial and error: y = 2x at x = 2.45 (accept 2.44 or 2.46) **[M1A1]**

[subtotal 4]

[TOTAL 24]

4(i)



[G2]

Two tps: could be cubic **[E1]**

Almost passes through origin **[E1]**

[subtotal 4]

(ii) $Q = \Sigma (y - ax - bx^2 - cx^3)^2$ **[M1]**

$\partial Q/\partial a = 0$ gives $\Sigma xy = a\Sigma x^2 + b\Sigma x^3 + c\Sigma x^4$ as given **[M1A1]**

other equations: $\Sigma x^2y = a\Sigma x^3 + b\Sigma x^4 + c\Sigma x^5$ **[B1]**

$\Sigma x^3y = a\Sigma x^4 + b\Sigma x^5 + c\Sigma x^6$ **[B1]**

[subtotal 5]

(iii) $\Sigma x^3 = 0$ $\Sigma x^5 = 0$

x	**y**	**x^2**	**x^4**	**x^6**	**xy**	**x^2y**	**x^3y**	
-3	-35.25	9	81	729	105.75	-317.25	951.75	
-2	-8.01	4	16	64	16.02	-32.04	64.08	
-1	2.51	1	1	1	-2.51	2.51	-2.51	
0	-0.09	0	0	0	0	0	0	
1	-4.07	1	1	1	-4.07	-4.07	-4.07	
2	-5.06	4	16	64	-10.12	-20.24	-40.48	
3	0.65	9	81	729	1.95	5.85	17.55	***totals***
		28	**196**	**1588**	**107.02**	**-365.24**	**986.32**	**[M1A3]**

Normal equations: $107.02 = 28a + 196c$

$-365.24 = 196b$

$986.32 = 196a + 1588c$ ***set up & solve***

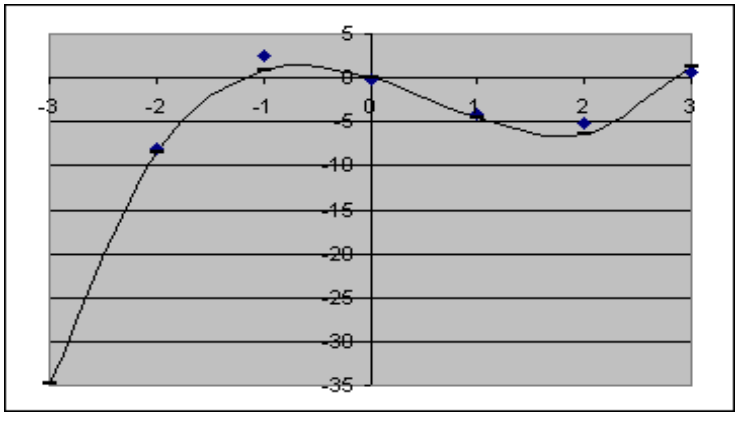
hence: $a = -3.86425$ $b = -1.86347$ $c = 1.098056$ **[M1A3]**

x	**y**	**y-fitted**	**res**	**res^2**
-3	-35.25	-34.826	0.424014	0.179788
-2	-8.01	-8.50983	-0.49983	0.24983
-1	2.51	0.902721	-1.60728	2.583345
0	-0.09	0	0.09	0.0081
1	-4.07	-4.62966	-0.55966	0.313219
2	-5.06	-6.39793	-1.33793	1.790044
3	0.65	1.283537	0.633537	0.40137
		-2.85714	**5.525696**	

[M1A1A1A1]

[subtotal 12]

(iv)



[G2]

Comment on goodness of fit **[E1]**

[subtotal 3]

[TOTAL 24]

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