OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI) 4764

Mechanics 4

Candidates answer on the answer booklet.

OCR supplied materials:
- 8 page answer booklet
  (sent with general stationery)
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Thursday 16 June 2011
Afternoon

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet. Please write clearly and in capital letters.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by g m s^{-2}. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

Section A (24 marks)

1 A raindrop of mass m falls vertically from rest under gravity. Initially the mass of the raindrop is m_0. As it falls it loses mass by evaporation at a rate λm, where λ is a constant. Its motion is modelled by assuming that the evaporation produces no resultant force on the raindrop. The velocity of the raindrop is v at time t. The forces on the raindrop are its weight and a resistance force of magnitude kmv, where k is a constant.

(i) Find m in terms of m_0, λ and t. **[2]**

(ii) Write down the equation of motion of the raindrop. Solve this differential equation and hence show that $v = \frac{g}{\lambda - k}\left(e^{(\lambda-k)t} - 1\right)$. **[8]**

(iii) Find the velocity of the raindrop when it has lost half of its initial mass. **[2]**

2 A small ring of mass m can slide freely along a fixed smooth horizontal rod. A light elastic string of natural length a and stiffness k has one end attached to a point A on the rod and the other end attached to the ring. An identical elastic string has one end attached to the ring and the other end attached to a point B which is a distance a vertically above the rod and a horizontal distance $2a$ from the point A. The displacement of the ring from the vertical line through B is x, as shown in Fig. 2.

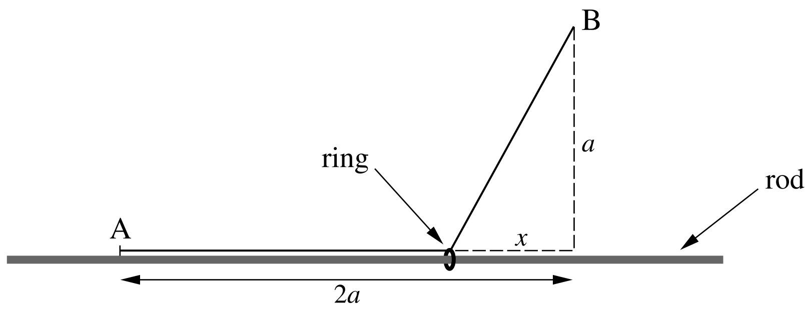

Fig. 2

(i) Find an expression for V, the potential energy of the system when $0 < x < a$, and show that

$$\frac{dV}{dx} = 2kx - ka - \frac{kax}{\sqrt{a^2 + x^2}}.$$ **[5]**

(ii) Show that $\frac{d^2V}{dx^2}$ is always positive. **[4]**

(iii) Show that there is a position of equilibrium with $\frac{1}{2}a < x < a$. State, with a reason, whether it is stable or unstable. **[3]**

Section B (48 marks)

3 A car of mass 800 kg moves horizontally in a straight line with speed $v\,\mathrm{m\,s^{-1}}$ at time t seconds. While $v \leqslant 20$, the power developed by the engine is $8v^4$ W. The total resistance force on the car is of magnitude $8v^2$ N. Initially $v = 2$ and the car is at a point O. At time t s the displacement from O is x m.

(i) Find v in terms of x and show that when $v = 20$, $x = 100 \ln 1.9$. **[10]**

(ii) Find the relationship between t and x, and show that when $v = 20$, $t \approx 19.2$. **[6]**

The driving force is removed at the instant when v reaches 20.

(iii) For the subsequent motion, find v in terms of t. Calculate t when $v = 2$. **[8]**

4 In this question you may assume without proof the standard results in *Examination Formulae and Tables (MF2)* for

- the moment of inertia of a disc about an axis through its centre perpendicular to the disc,
- the position of the centre of mass of a solid uniform cone.

Fig. 4 shows a uniform cone of radius a and height $2a$, with its axis of symmetry on the x-axis and its vertex at the origin. A thin slice through the cone parallel to the base is at a distance x from the vertex.

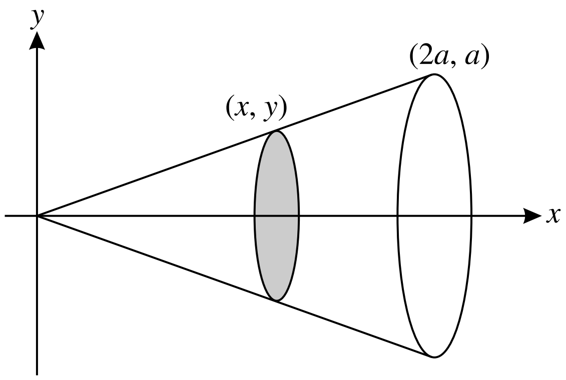

Fig. 4

The slice is taken to be a thin uniform disc of mass m.

(i) Write down the moment of inertia of the disc about the x-axis. Hence show that the moment of inertia of the disc about the y-axis is $\frac{17}{16}mx^2$. **[6]**

(ii) Hence show by integration that the moment of inertia of the cone about the y-axis is $\frac{51}{20}Ma^2$, where M is the mass of the cone. [You may assume without proof the formula for the volume of a cone.] **[8]**

The cone is now suspended so that it can rotate freely about a fixed, horizontal axis through its vertex. The axis of symmetry of the cone moves in a vertical plane perpendicular to the axis of rotation. The cone is released from rest when its axis of symmetry is at an acute angle α to the downward vertical. At time t, the angle the axis of symmetry makes with the downward vertical is θ.

(iii) Use an energy method to show that $\dot{\theta}^2 = \dfrac{20g}{17a}(\cos\theta - \cos\alpha)$. **[5]**

(iv) Hence, or otherwise, show that if α is small the cone performs approximate simple harmonic motion and find the period. **[5]**

THERE ARE NO QUESTIONS PRINTED ON THIS PAGE.