GCE

Mathematics (MEI)

Advanced GCE

Unit **4767:** Statistics 2

Mark Scheme for June 2011

Subject-specific Marking Instructions for GCE Mathematics (MEI) Statistics strand

a   Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b   An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c   The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d   When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e   The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f   Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

Candidates are expected to give numerical answers to an appropriate degree of accuracy. 3 significant figures may often be the norm for this, but this always needs to be considered in the context of the problem in hand. For example, in quoting probabilities from Normal tables, we generally expect *some* evidence of interpolation and so quotation to 4 decimal places will often be appropriate. But even this does not always apply – quotations of the standard critical points for significance tests such as 1.96, 1.645, 2.576 (maybe even 2.58 – but not 2.57) will commonly suffice, especially if the calculated value of a test statistic is nowhere near any of these values. Sensible discretion *must* be exercised in such cases.

Discretion must also be exercised in the case of small variations in the degree of accuracy to which an answer is given. For example, if 3 significant figures are expected (either because of an explicit instruction or because the general context of a problem demands it) but only 2 are given, loss of an accuracy ("A") mark is likely to be appropriate; but if 4 significant figures are given, this should not normally be penalised. Likewise, answers which are slightly deviant from what is expected in a very minor manner (for example a Normal probability given, after an attempt at interpolation, as 0.6418 whereas 0.6417 was expected) should not be penalised. However, answers which are *grossly* over- or under-specified should normally result in the loss of a mark. This includes cases such as, for example, insistence that the value of a test statistic is (say) 2.128888446667 merely because that is the value that happened to come off the candidate's calculator. Note that this applies to answers that are given as final stages of calculations; intermediate working should usually be carried out, and quoted, to a greater degree of accuracy to avoid the danger of premature approximation.

The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g   Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h   Genuine misreading (of numbers or symbols, occasionally even of text) occurs. If this results in the object and/or difficulty of the question being considerably changed, it is likely that all the marks for that question, or section of the question, will be lost. However, misreads are often such that the object and/or difficulty remain substantially unaltered; these cases are considered below.

The simple rule is that *all* method ("M") marks [and of course all independent ("B") marks] remain accessible but at least some accuracy ("A") marks do not. It is difficult to legislate in an overall sense beyond this global statement because misreads, even when the object and/or difficulty remains unchanged, can vary greatly in their effects. For example, a misread of 1.02 as 10.2 (perhaps as a quoted value of a sample mean) may well be catastrophic; whereas a misread of 1.6748 as 1.6746 may have so slight an effect as to be almost unnoticeable in the candidate's work.
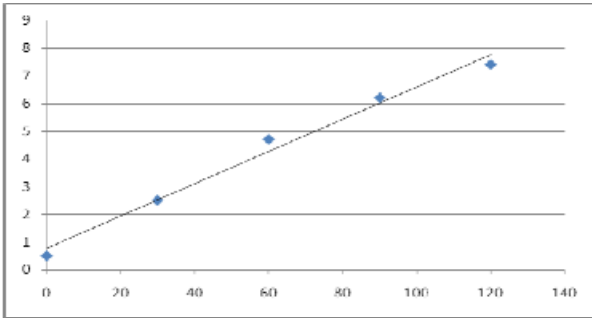
A misread should normally attract *some* penalty, though this would often be only 1 mark and should rarely if ever be more than 2. Commonly in sections of questions where there is a numerical answer either at the end of the section or to be obtained and commented on (eg the value of a test statistic), this answer will have an "A" mark that may actually be designated as "cao" [correct answer only]. This should be interpreted *strictly* – if the misread has led to failure to obtain this value, then this "A" mark must be withheld even if all method marks have been earned. It will also often be the case that such a mark is implicitly "cao" even if not explicitly designated as such.

On the other hand, we commonly allow "fresh starts" within a question or part of question. For example, a follow-through of the candidate's value of a test statistic is generally allowed (and often explicitly stated as such within the marking scheme), so that the candidate may exhibit knowledge of how to compare it with a critical value and draw conclusions. Such "fresh starts" are not affected by any earlier misreads.

A misread may be of a symbol rather than a number – for example, an algebraic symbol in a mathematical expression. Such misreads are more likely to bring about a considerable change in the object and/or difficulty of the question; but, if they do not, they should be treated as far as possible in the same way as numerical misreads, *mutatis mutandis*. This also applied to misreads of text, which are fairly rare but can cause major problems in fair marking.

The situation regarding any particular cases that arise while you are marking for which you feel you need detailed guidance should be discussed with your Team Leader.
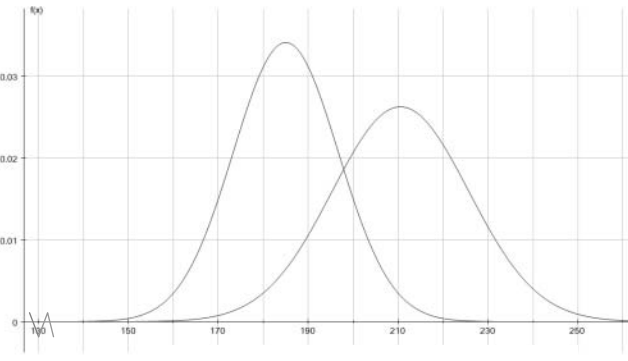
Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

1 (i)		G1 for axes G1 For values of x G1 for values of y		Condone axes drawn either way. Axes should show some indication of scale. If not then Max G1 if points 'visibly correct'. If axes are scaled and only one point is incorrectly plotted, allow max G2/3.
			3	
1 (ii)	$\bar{x} = 60,\ \bar{y} = 4.26$ $b = \dfrac{S_{xy}}{S_{xx}} = \dfrac{1803 - 300 \times 21.3/5}{27000 - 300^2/5} = \dfrac{525}{9000} = 0.0583$ OR $\quad b = \dfrac{1803/5 - 60 \times 4.26}{27000/5 - 60^2} = \dfrac{105}{1800} = 0.0583$ hence least squares regression line is: $\qquad y - \bar{y} = b(x - \bar{x})$ $\Rightarrow \quad y - 4.26 = 0.0583\,(x - 60)$ $\Rightarrow \qquad y = 0.0583x + 0.76$	B1 for $\bar{x}$ and $\bar{y}$ used appropriately (SOI) M1 for attempt at gradient (b) A1 for 0.0583 **cao** M1 for equation of line A1 FT for complete equation		B1 for means can be implied by a correct value of b using either method. Allow $\bar{y} = 4.3$ Attempt should be correct – e.g. evidence of either of the two suggested methods should be seen. Allow 0.058 Condone $0.058\dot{3}$ and $\dfrac{7}{120}$ Dependent on first M1. Values must be substituted to earn M1. Condone use of their b for FT provided $b>0$. Final equation must be simplified.
			5	$b = 0.058$ leads to $y = 0.058x + 0.78$
1 (iii)	Regression line plotted on graph The fit is good	G1 G1 E1 for good fit		Line must pass through their $(\bar{x}, \bar{y})$ and y-intercept. E0 for notably inaccurate graphs/lines
			3	

1 (iv)	$x = 30 \Rightarrow$ predicted $y = 0.0583 \times 30 + 0.76 = 2.509$ Residual = 2.5 − 2.509 = −0.009	B1 for prediction M1 for subtraction A1 FT	**3**	Using their **equation** Subtraction can be 'either way' but for the final mark the sign of the residual must be correct. FT sensible equations only – e.g. no FT for $y = 0.071x$ leading to +0.37. [$c = 0.78$ leads to a residual of -0.02]

1 (v)	(*A*) For $x = 45$,
	             $y = 0.0583 \times 45 + 0.76 \quad = 3.4$
	
	 (*B*) For $x = 150$,
	             $y = 0.0583 \times 150 + 0.76 \quad = 9.5$
		M1 for at least one prediction attempted
		
		 A1 for **both answers** (FT their equation provided their $b > 0$)
			2	Prediction obtained from their equation.

1 (vi)	This is **well** below the predicted value … …suggesting that the model breaks down for larger values of x.	E1 for **well** below E1 extrapolation	**2**	Some indication that the value (8.7) is **significantly** below what is expected (9.5) is required for the first E1. Simply pointing out that it is 'below' is not sufficient. The second E1 is available for a suitable comment relating to the model being suitable only for values within the domain of the given points. Allow other sensible comments for either E1. E.g. The data might be better modelled by a curve', 'there may be other factors affecting yield',
			18	

2 (i)	Independently means that the arrival time of each car is unrelated to the arrival time of any other car. Randomly means that the arrival times of cars are not predictable. At a uniform average rate means that the average rate of car arrivals does not vary over time.	E1 E1 E1	**3**	NOTE **Each answer must be 'in context' and 'clear'** Allow sensible alternative wording. SC1 For ALL answers not in context but otherwise correct.
2 (ii)	$P(\text{At most 1 car}) = e^{-0.62}\dfrac{0.62^0}{0!} + e^{-0.62}\dfrac{0.62^1}{1!}$ $= 0.5379\ldots + 0.3335\ldots = 0.871$	M1 for either M1 for sum of both A1 CAO	**3**	$1.62e^{-0.62}$ Allow 0.8715 not 0.872 or 0.8714 Allow 0.87 without wrong working seen
2 (iii)	New $\lambda = 10 \times 0.62 = 6.2$ P(more than 5 in 10 mins) $= 1 - 0.4141 = 0.5859$	B1 for mean (SOI) M1 for probability A1 CAO	**3**	Use of $1 - P(X \le 5)$ with any λ Allow 0.586
2 (iv)	Poisson with mean 37.2	B1 for Poisson B1 for mean 37.2	**2**	Dependent on first B1 Condone P(37.2, 37.2)
2 (v)	Use Normal approx with $\mu = \sigma^2 = \lambda = 37.2$ $P(X \ge 40) = P\left(Z > \dfrac{39.5 - 37.2}{\sqrt{37.2}}\right)$ $= P(Z > 0.377) = 1 - \Phi(0.377) = 1 - 0.6469$ $= 0.3531$	B1 for Normal (SOI) B1 for parameters B1 for 39.5 M1 for correct use of Normal approximation using correct tail A1 **cao**	**5**	Allow 0.353
			16	

3 (i)	P(Apple weighs at least 220g) $$= P\left(Z > \frac{220 - 210.5}{15.2}\right)$$ $$= P(Z > 0.625)$$ $$= 1 - \Phi(0.625) = 1 - 0.7340$$ $$= 0.2660$$	M1 for standardising M1 for correct structure A1 CAO inc use of diff tables	**3**	Condone numerator reversed but penalise continuity corrections i.e. $1 - \Phi$(positive z value) Allow 0.266 but not 0.27
3 (ii)	P(All 3 weigh at least 220g) $= 0.2660^3 = 0.0188$	M1 A1 FT	**2**	M1 for their answer to part (i) cubed Allow 0.019 and 0.01882
3 (iii)	(A) Binomial (100, 0.0188) (B) Use a Poisson distribution with $\qquad \lambda = 100 \times 0.0188 = 1.88$ $$P(\text{At most one}) = e^{-1.88}\frac{1.88^0}{0!} + e^{-1.88}\frac{1.88^1}{1!}$$ $$= 0.1525 + 0.2869 = 0.4394$$ (C) n is large and p is small	B1 for binomial B1 for parameters B1 for Poisson SOI B1 for Poisson mean M1 for either probability M1 for sum of both A1 CAO For 0.44 or better B1	**2** **5** **1**	Second B1 dependent on first B1 FT their answer to part (ii) for second B1 Consistent with $p < 0.1$ from part (iii) (A) FT answer to part (ii) with $p < 0.1$ Dependent on both previous B1 marks Allow 0.4395 but not 0.4337 Dependent on use of Poisson in part (iii) B Allow n is large and $np < 10$ & n is large and $np \approx npq$
3(iv)(A)	$\Phi^{-1}(0.1) = -1.282$ $$\frac{170 - 185}{\sigma} = -1.282$$ $1.282\,\sigma = 15$ $\sigma = 11.70$	B1 for ± 1.282 M1 for correct equation as written o.e. A1 CAO	**3**	Do not allow $1 - 1.282$ Allow M1 if different **z-value** used Without incorrect working seen. Allow 11.7

3(iv)(*B*)	Cox's Braeburns	G1 for shape G1 for means, shown explicitly or by scale G1 for lower max height for Braeburns G1 for greater width (variance) for Braeburns	**4**	Ignore labelling of vertical axis. Two intersecting, adjacent Normal curves Means at 185 and 210.5
		TOTAL	**20**	
4(a)(i)	H_0: no association between amount spent and sex H_1: some association between amount spent and sex	B1 for both	**1**	Hypotheses must be the right way round, in context and must not mention 'correlation'.
4(a)(ii)	Expected frequency = $62 \times 102 \div 200 = 31.62$ Contribution = $(34 - 31.62)^2 / 31.62$ $= 0.1791$	B1 M1 A1 for valid attempt at $(O-E)^2/E$ ***NB Answer given***	**3**	Do not allow 31.6

4(a)(iii)	Refer to X_4^2 Critical value at 5% level = 9.488 3.205 < 9.488 Result is not significant There is insufficient evidence to suggest any association between amount spent and sex. .	B1 for 4 deg of freedom B1 CAO for cv M1 A1 for not significant E1		Allow $p = 0.524$ 0.524 > 0.05 Conclusion must be stated to earn A1 here. Allow 'do not reject H_0' and condone 'accept H_0' or 'reject H_1'. FT if cv consistent with their d.o.f. Dependent on previous A1 and final comment must be in context and not mention correlation. SC1 for correct final conclusion where previous A1 omitted but M1 awarded.
			5	
4 (b)	H_0: $\mu = 400$; H_1: $\mu < 400$ Where μ denotes the population mean (weight of the loaves). $\bar{x} = 396.5$ Test statistic $= \dfrac{396.5 - 400}{5.7/\sqrt{6}} = \dfrac{-3.5}{2.327} = -1.504$ 5% level 1 tailed critical value of $z = -1.645$ $-1.504 > -1.645$ so not significant. There is insufficient evidence to reject H_0 There is insufficient evidence to suggest that the true mean weight of the loaves is lower than the minimum specified value of 400 grams.	B1 for H_0 B1 for H_1 B1 for definition of μ B1 for sample mean M1 must include $\sqrt{6}$ A1FT their sample mean B1 for ±1.645 M1 for sensible comparison leading to a conclusion A1 for conclusion in context		Hypotheses in words must refer to population mean. Condone numerator reversed for M1 but award A1 only if test statistic of 1.504 is compared with a positive z-value. Dependent on previous M1 FT their sample mean only if hypotheses are correct.
			9	
		TOTAL	**18**	

Additional notes re Q1 parts (ii), (iv) and (v)

Part (ii) 'x on y' max B1

Part (iv) $x = 16.9y - 12.02$ leads to a prediction of $x = 30.23$ and a residual of -0.23 B1 M1 A1 available.

Part (v) 'x on y' not appropriate here so award 0 if 'x on y' used.

Additional notes re Q2 parts (i) & (v)

Part (i)

Independent – Allow 'not linked to' or 'no association' or 'unrelated' 'not affected by', 'not connected to', 'not influenced by'
DO NOT ACCEPT 'not together' or 'not dependent'

Random – Allow 'not predictable' or 'no pattern' or 'could happen at any time' or 'not specific time'

Uniform average rate – Allow 'average (rate) is constant over time' DO NOT ACCEPT 'average constant' or 'average rate and uniform' – be generous over defining 'average' and/or 'rate'.

Part (v) If Binomial distribution stated in part (iv), allow B1 B0 B1 M0 A0 max

Additional notes re Q3 part (iii) where $p > 0.1$

(iii) B – as scheme unless a Normal approximation is more suitable ($p > 0.1$). If so, award B1 B1 for Normal and correct parameters. The remaining marks are dependent on both these B1 marks being awarded. M1 for the correct continuity correction (P($X < 1.5$)) and depM1 for the correct tail but award A0.

(iii) C – 'n is large and p is not too small' or '$np > 10$'

Additional notes re Q4(b)

<u>σ estimated</u>
sample mean, 7.079… used in place of 5.7, the given value of the population mean, leads to a test statistic of -1.212… This gets M1A0 & the remaining marks are still available.

<u>Critical Value Method</u>
400 – 1.645 × 5.7 ÷ √6… gets M1B1 …= 396.17…gets A1
400 + 1.645 × 5.7 ÷ √6 gets M1B1A0.
396.5 > 396.2 gets M1for sensible comparison (and B1 for 396.5)
A1 still available for correct conclusion in words & context

<u>90% Confidence Interval Method</u>
CI centred on 396.5 (gets B1 for 396.5)
+ or − 1.645 × 5.7 ÷ √6 gets M1 B1
= (392.67, 400.33) A1
contains 400 gets M1
A1 still available for correct conclusion in words & context

<u>Probability Method</u>
Finding P(sample mean < 396.5) = 0.0663 gets M1 A1 (and B1 for 396.5)
0.0663 > 0.05 gets M1 for a sensible comparison if a conclusion is made and also gets the B1 for 0.0663 (to replace the B1 for cv = 1.645).
A1 still available for correct conclusion in words & context.
Condone P(sample mean >396.5) = 0.9337 for M1 and B1 for 0.9337 but only allow A1 if later compared with 0.95 at which point the final M1and A1 are still available

<u>Two-tailed test</u>
Max B1 B0 B1 B1 M1 A1 B1 (for cv = -1.96) M1 A0