OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE

MATHEMATICS (MEI)

Numerical Methods

4776/01

Candidates answer on the answer booklet.

OCR supplied materials:
- 8 page answer booklet (sent with general stationery)
- Graph paper
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Friday 14 January 2011
Afternoon

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet. Please write clearly and in capital letters.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Do **not** write in the bar codes.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

Section A (36 marks)

1 **(i)** Show that the equation $1 + x = \tan x$, where x is in radians, has a root in the interval [1, 1.2]. **[2]**

(ii) Show numerically that the iteration $x_{r+1} = \tan x_r - 1$ with $x_0 = 1.1$ diverges. **[2]**

(iii) Use another iteration to find the root correct to 3 decimal places. **[4]**

2 The table shows some estimates of an integral, $\int_2^4 \mathrm{f}(x)\,\mathrm{d}x$, using the mid-point rule (M) and the trapezium rule (T), for given values of h.

h	M	T
2	1.987 467	1.354 440
1	1.830 595	
0.5		

Copy the table and fill in the additional estimates that can be found.

Obtain the Simpson's rule estimates that can be found.

Give the value of the integral to the accuracy that appears justified. **[8]**

3 The table shows values of $\mathrm{g}(x)$ correct to 4 decimal places.

x	0	0.5	1
$\mathrm{g}(x)$	1.4509	1.6799	2.0100

(i) Use the forward difference method to find two estimates of $\mathrm{g}'(0)$. State, with a reason, which of these is likely to be more accurate. **[4]**

(ii) Use the central difference method to find an estimate of $\mathrm{g}'(0.5)$. Comment on the likely accuracy of this estimate compared to those in part **(i)**. **[2]**

4 A bank's computer system calculates the interest payable on each savings account every day. A running total is kept of the daily amounts of interest, and accounts are credited with this interest at the end of each year. The bank used to *round* the daily amounts of interest payable to the nearest 0.01 of a penny, but they decide to *chop* to the nearest 0.01 of a penny instead.

(i) Find the maximum possible loss in a year to a savings account because of the chopping, and explain how this loss could occur. State, with a reason, what the average loss will be. **[4]**

(ii) The bank calculates that chopping in this way will generate an additional profit of about £150 000 per year. Estimate the number of savings accounts the bank has. **[2]**

5 The function P(x) is known to be a polynomial. Some values of P(x) are given in the table.

x	1	3	5	7	9
P(x)	–10	3	44	129	274

(i) Use a difference table to determine, with a reason, the least possible degree of polynomial that will fit all the data points. **[4]**

(ii) Assuming that P(x) is of this degree, extend your table to find the values of P(–1) and P(11). **[4]**

Section B (36 marks)

6 In this question,

$$f(x) = \frac{x}{\sin x} - \frac{\sin x}{x},$$

where x is in radians. For small non-zero values of x, f(x) may be approximated by g(x) or by h(x), where

$$g(x) = \frac{1}{3}x^2 \qquad \text{and} \qquad h(x) = \frac{2x^2}{6 - x^2}.$$

(i) Find the absolute and relative errors in g(x) and h(x) as approximations to f(x) for

(A) $x = 0.2$,

(B) $x = 0.1$ **[9]**

(ii) A third approximation to f(x) is given by $\frac{4g(x) + h(x)}{5}$. Explain by reference to part **(i)** why this would be expected to be a good approximation.
Find the absolute and relative errors when this third approximation is used to estimate f(0.2) and f(0.1). **[6]**

(iii) Use your calculator to evaluate $\frac{x}{\sin x}$ when $x = 10^{-4}$.

When $x = 10^{-4}$, a cheap calculator evaluates f(x) as zero. Use an approximate formula to find a better value for f(10^{-4}). Explain why the cheap calculator makes an error. **[3]**

7 **(i)** Show that the equation $f(x) = 0$, where

$$f(x) = x^7 + x^5 - 1, \qquad \textbf{(*)}$$

has a root in the interval [0, 1].
By considering $f'(x)$ show that there are no other roots.
Sketch the graph of $y = f(x)$ for $x \geqslant 0$. **[7]**

(ii) Obtain the Newton-Raphson iteration based on (*). Starting with $x_0 = 0.6$, find x_1 and x_2. Illustrate this iteration on your sketch of $y = f(x)$. **[7]**

(iii) Use the Newton-Raphson iteration to find x_1 and x_2 in the cases

(*A*) $x_0 = 0.3$,

(*B*) $x_0 = 0.9$.

Comment on your results in each case. **[4]**